CENTER FOR TECHNOLOGY IN LEARNING

TI-Nspire™ Math and Science Learning Handhelds: What Research Says and What Educators Can Do

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The purpose of this document is to suggest a framework which educators and researchers may find helpful in their efforts to improve teaching practices and mathematics achievement. SRI International does not endorse or recommend specific products.

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Executive Summary

With the release of TI-Nspire™ math and science learning handhelds and software, Texas Instruments offers a learning platform with augmented capabilities that extend its traditional graphing calculator line. We anticipate that educators and researchers will look at this new platform as an opportunity to advance mathematics teaching practice and student learning outcomes. The platform includes both new features on the handheld device as well as the classroom networking capabilities: it is a platform for calculating, representing and communicating mathematically. One key new capability is that students work in a "document", an organized presentation of multiple screens of mathematics which can be saved, shared, annotated, and revisited. We suggest a framework which educators and researchers may find helpful in articulating the research basis for their efforts to improve teaching practices and mathematics achievement.

We suggest that teachers think of the research basis for TI-Nspire use in terms of three layers:

1. **Effectiveness.** TI-Nspire builds on and unites two strong research findings: Graphing calculators enhance student learning. Incorporating formative assessment into everyday teaching practice is highly effective. When integrating TI-Nspire learning handhelds into their practice, teachers can draw insights from a rich literature substantiating effective use of graphing calculators and formative assessment in mathematics and science classrooms.

2. **Enhanced representation and communication of important mathematics.** TI-Nspire's linked representations should help teachers to focus students' attention on the relationships among multiple representations, such as algebraic equations, geometric constructions, graphs, and tables of data. Networking capabilities can increase student participation and engage students in mathematical thinking and communication around these representations. TI-Nspire's multiple representation and communication capabilities can make thinking visible and can support the classroom teacher to engage students in doing and discussing important mathematics.

3. **Deeper opportunities to learn.** Using the new document and networking features of TI-Nspire, teachers can develop classroom practices that increase the time students spend doing mathematics in an environment that has the ingredients for success: increased support for mastering difficult concepts and skills; high student participation; and tools for reflective practice.

The opportunity to draw upon research support at these three levels should be exciting for educators and scholars who are looking for new ways to improve mathematics learning, especially as educators begin to draw upon the benefits of having capabilities that extend beyond the familiar graphing calculator. For example, teachers will have new opportunities to differentiate instruction. Additional instructional models will allow teachers to support project-based learning, engage in participatory simulations, and encourage students to build mathematical models. Teachers who use the new communication capabilities to engage students in more active participation may achieve increased attendance, less disciplinary problems and a more cooperative classroom environment. Overall, educators and scholars may use TI-Nspire handhelds as their vehicle for exploring new ways to give every student an engaging, inspiring, and successful learning experience.

Introduction

With the release of TI-Nspire™ math and science learning handhelds, Texas Instruments offers a learning platform with augmented capabilities that extend its traditional graphing calculator line. We anticipate that educators and researchers will look at this new platform as an opportunity to advance mathematics teaching practice and student learning outcomes. The platform includes both new features on the handheld device as well as the classroom networking capabilities: it is a platform for calculating, representing and communicating mathematically. The features available on the handheld device are also available via software that can be installed on a laptop or desktop computer. One key new capability is that students work in a "document", an organized presentation of multiple screens of mathematics which can be saved, shared, annotated, and revisited. The design of this platform, including the new document capability, has been informed by and aligns with research principles. We suggest a framework which educators and researchers may find helpful in articulating the research basis for their efforts to improve teaching practices and mathematics achievement.

We suggest thinking of the research basis for the use of TI-Nspire in terms of three layers:

1. **Effectiveness.** TI-Nspire builds on and unites two strong research findings: Graphing calculators enhance student learning. Incorporating formative assessment into everyday teaching practice is highly effective. When integrating TI-Nspire learning handhelds into their practice, teachers can draw insights from a rich literature base substantiating effective use of graphing calculators and formative assessment in mathematics and science classrooms.

2. **Enhanced representation and communication of important mathematics.** TI-Nspire's linked representations should help teachers to focus students' attention on the relationships among multiple representations, such as algebraic equations, geometric constructions, graphs, and tables. Networking capabilities can increase student participation and engage students in mathematical thinking and communication around these representations. TI-Nspire's multiple representation and communication capabilities can make thinking visible and can support the classroom teacher to engage students in doing and discussing important mathematics.

3. **Deeper opportunities to learn.** Using the new document features of TI-Nspire, teachers can develop classroom practices that increase the time students spend doing mathematics in an environment that has the ingredients for success: increased support for mastering difficult concepts and skills; high student participation; and tools for reflective practice.

We discuss each layer of support in separate sections below. In each section, we describe both what research says and what teachers can do.

How TI-Nspire™ Can Enable Better Teaching

What Research Says	What Mathematics Teachers Can Do
Effectiveness	
Graphing calculator use increases student achievement. In particular, students who use calculators daily or weekly learn more.	Use TI-Nspire's calculator, algebra, graphing, table, and geometry features to: 1. Emphasize problem solving and concepts, not just right answers. 2. Engage students with interactive exploration. 3. Focus students who have mastered underlying calculations on mathematical concepts, strategies and justifications.
Formative assessment increases student achievement.	Use TI-Nspire's ability to involve all students in a quick assessment to: ▪ Pose questions that stimulate student reasoning and explanation. ▪ See what students are thinking and doing to help them improve. ▪ Give students feedback more often and more quickly than is possible through graded homework. ▪ Gauge whether the pace of instruction is too fast or too slow.
Enhanced representation and communication of important mathematics	
Linked multiple representations (e.g., equations, graphs, tables, geometric sketches, words) enable students to master difficult concepts.	Use TI-Nspire's clear and expressive representations to: 1. Reinforce the meaning of a representation (e.g., what each axis in a graph represents). 2. Focus attention on the same mathematical idea across representations. 3. Explore the effects of changing variables across representations. 4. Integrate geometric tools into the teaching of graphical concepts (e.g., construct a rectangular area under a curve). 5. Introduce topics, such as modeling, that were previously too time consuming.
Classroom networking engages students in doing and communicating important mathematics.	Use TI-Nspire with TI-Navigator's classroom network to: 1. Accelerate student thinking by sending mathematical ideas, models and tasks to students and rapidly collect their work for discussion. 2. Allow students to communicate with multiple representations—words, equations, graphs, or geometric sketches. 3. Base classroom discussions on student work and, through attention to student work, show that mathematical communication is valued.

How TI-Nspire™ Can Enable Better Teaching	
What Research Says	**What Mathematics Teachers Can Do**
Deeper opportunities to learn	
Students learn more when they have more **academic learning time**, tasks are neither too easy nor too hard, and they experience early and frequent success.	Use documents and a classroom network to: 1. Prepare a context for student work so they can start mathematical work quickly. 2. Organize a sequence of tasks so students succeed early and frequently (e.g., differentiated instruction). 3. Send a common starting point (e.g., a function and graph zoom settings) to all students, thus "getting on the same page" instantly. 4. Reduce the class time spent distributing and collecting assignments.
Students learn complex knowledge and skills best when teachers provide **scaffolding**—supports and resources that fade as students gain mastery.	Use documents and a classroom network to: 1. Provide worked examples for students to use as models. 2. Demonstrate the steps or phases in a mathematical investigation. 3. Examine the connections between and across rich mathematical problems.
Students can help each other to learn and collaborate to improve ideas; **participatory activities** are a powerful resource for accelerating learning.	Use documents and a classroom network to: 1. Help students to work together productively, for example, by building on each others' mathematical ideas, offering constructive critiques, and taking complementary roles in a shared project. 2. Encourage students to collaborate on a presentation of a shared solution to a complex problem. 3. Develop complex mathematical ideas through participatory simulations—shared simulations in which each student plays a unique role in the group experience of a mathematical phenomenon. 4. Involve students in constructing a mathematical narrative together—a shared mathematical performance in which students must coordinate their mathematical contributions to achieve a shared goal.
Reflection and **revision** are at the heart of learning—and also learning to be a better learner.	Use documents and a classroom network to: 1. Compare a prediction to what happens in an exploration. 2. Aggregate students' individual work as the basis for group discussions. 3. Explore generalizations across different students' solutions. 4. Create opportunities to revise and improve students' mathematics.

Effectiveness

TI-Nspire™ builds on and aligns with two strong research findings: Graphing calculators enhance student learning. Incorporating formative assessment into everyday teaching practice is highly effective. When integrating TI-Nspire learning handhelds into their practice, teachers can draw insights from a rich literature base substantiating effective use of graphing calculators and formative assessment in mathematics and science classrooms. We discuss both the graphing calculator and formative assessment research bases below.

In mathematics, graphing products are integrated in national and many state standards (e.g., Texas Legislature 1998; National Council of Teachers of Mathematics 2000) and supported in curricula. Best practices of instruction are well-documented (Burrill et al., 2002) and teacher professional development offerings are widely available. Using graphing products, teachers can enhance their classroom by:

- Increasing the attention to conceptual understanding and problem solving strategies by providing computational support and spending less class time on laborious computations
- Examining the related meanings of a concept through display of multiple representations, such as exploring rate of change in a graph (i.e., slope) and table
- Engaging students with interactive explorations, real world data collection, and more authentic data sets
- Giving students more responsibility for checking their work and justifying their solutions
- Introducing topics that were previously too difficult for many students (e.g., modeling)

In this section, we summarize the research on graphing calculators across a spectrum of data sets.

Experiments in which students are randomly assigned to control and treatment groups are the gold standard for educational research. A large number of such studies has been conducted. When many experimental studies have been performed, researchers summarize the results via a meta-analysis, which provides policymakers with a robust estimate of an intervention's true effectiveness. A meta-analysis by Ellington (2003) reviewed an inclusive set of 54 high-quality experimental studies. This meta-analysis shows a reliable positive effect of graphing calculator-based interventions upon student achievement. In addition, the studies suggest that when graphing calculators are allowed on tests, gains extend from calculations and operations to conceptual understanding and problem solving performance. A second meta-analysis looked specifically at algebra. Khoju, Jaciw, & Miller (2005) screened available research using stringent quality-control criteria published by the U.S. Department of Education's What Works Clearinghouse. They selected eight high-quality studies examining the impact of graphing calculators on K-12 mathematics achievement. Four of these studies specifically assessed the impact on algebra learning. Across a wide variety of student populations and teaching conditions, use of graphing calculators with aligned instructional materials was shown to have a strong, positive effect on algebra achievement.

Educators and policymakers often have questions beyond "what works"—they want concrete guidance on how to achieve an effective implementation and confidence that if they implement at scale, they will see positive results. Strong graphing calculator research is available to address these concerns. A study by Heller (2005) described and studied a model implementation, which included a new textbook, teacher professional development, and assessment—all aligned with the graphing technology by the theme of "Dynamic Algebra." This study establishes that the teachers and students who used graphing calculators most frequently learned the most. The National Center for Educational Statistics signature report, "The Nation's Report," (National Center for Education Statistics, 2001, p. 144) provides confidence that frequent use at the eighth grade level (but not at the fourth grade level) is associated with greater mathematics achievement, stating:

> Eighth-graders whose teachers reported that calculators were used almost every day scored highest. Weekly use was also associated with higher average scores than less frequent use. In addition, teachers who permitted unrestricted use of calculators and those who permitted calculator use on tests had eighth-graders with higher average scores than did teachers who did not indicate such use of calculators in their classrooms. The association between frequent graphing calculator use and high achievement holds for both richer and poorer students, for both girls and boys, for varied students with varied race and ethnicity, and across states with varied policies and curricula.

In summary, the evidence for the impact of calculator use on student achievement is robust and consistent. Ellington's (2003) summary of 54 studies includes a wide variety of grade levels, socioeconomic background, geography, and specific mathematical content. More than 80% of those 54 studies employed some form of random assignment to calculator use; this is the strongest research design known. The effects (particularly when calculators are used for both instruction and assessment) are substantial, often increasing an average student's achievement by 10 to 20 percentile points. In examining the impact of graphing calculators on algebra achievement in particular, Khoju, Jaciw, & Miller (2005) found even stronger effects. Lastly, all of these results are corroborated by consistent findings from the National Assessment of Educational Progress (NAEP), which reports higher achievement for students who use calculators frequently, drawing from a nationally representative sample of students (NCES, 2001).

Formative assessment draws upon another but equally strong research base. Formative assessment stems from the idea that providing constructive feedback early in the learning process can improve student achievement. A former president of the American Educational Research Association wrote:

> In order for assessment to play a more useful role in helping students learn it should be moved into the middle of the teaching and learning process instead of being postponed as only the end-point of instruction. Dynamic assessment…is integral to Vygotsky's idea of a zone of proximal development. This type of interactive assessment, which allows teachers to provide assistance as part of assessment, does more than help teachers gain valuable insights about how understanding might be extended. It also creates perfectly targeted occasions to teach and provides the means to scaffold next step. (Shepard, 2000, p. 10).

It is important to realize that formative assessment is not just conventional testing: it can happen on any occasion during which students "make thinking visible" (Bransford, Brown, & Cocking, 2000, p. 220) and teachers offer comments that give students direction on how to improve.

Because many new technologies are interactive (Greenfield & Cocking, 1996), it is now possible to create environments in which students can learn by doing, receive feedback, and continually refine their understanding and build new knowledge (Scardemalia & Bereiter, 1993). Assessment and learning can be intimately linked in technology-rich learning environments.

When teachers' questions are used to deepen students' higher-order thinking and when feedback is provided to students on how they can improve, significant learning gains can occur (Dillon & Wittrock, 1984; Gall, 1984; Redfield & Rousseau, 1981; Samson, Strykowski, Weinstein, & Walberg, 1987). In addition, higher achievement levels are reported in classrooms where students are involved in checking their own understanding of concepts and assessment data are used to inform and adjust instruction (Black & Wiliam, 1998; Fuchs & Fuchs, 1986). Meta-analytic studies and research reviews have shown that better classroom assessment results in significant gains in learning outcomes. In their review of 21 separate classroom assessment studies, for example, Fuchs and Fuchs (1986) found that the systematic use of data from formative assessments can dramatically improve learning outcomes. The researchers found that formative evaluation practices had a strong effect on achievement , when compared to control condition in 21 separate studies. Put differently, one could predict that teachers' use of formative assessment data to adjust instruction would raise the typical achievement outcome scores on a norm-referenced test from the 50th percentile to the 76th percentile. The conclusions of Black and Wiliam (1998, p.), based on their review of more than 250 separate studies and meta-analyses of a wide range of classroom assessment practices, are similar to those of Fuchs and Fuchs. Their synthesis found large effects for classroom assessment on student learning, effects greater than for nearly any other type of educational intervention:

> For public policy toward schools, the case to be made here is firstly that significant learning gains lie within our grasp. The research reported here shows conclusively that formative assessment does improve learning. The gains in achievement appear to be quite considerable, and as noted earlier, amongst the largest ever reported for educational interventions. (Black and Wiliam 1998)

TI-Nspire does not propose a one size-fits-all formative assessment solution, but it is designed to integrate with teacher's formative assessment practices. Teachers can use TI-Nspire learning handhelds to enhance formative assessment in at least three ways. First, TI-Nspire documents "make thinking visible." Whereas before students had to copy their work onto paper in order to show their work, now they can capture and present how they arrived at a problem's solution in the document format and share it with the classroom using the classroom network. Even more importantly, the early phases of mathematical exploration can now be preserved and discussed, not just "answers" and "solutions." Thus documents provide a basis for teachers to ask better questions. Second, teachers can now prepare assessments by writing a test as a document, enabling the tests to more easily incorporate technology capabilities, such as multiple representations and scaffolding. Even simple tests can be used as a "quick poll" that checks student understanding, with results instantly aggregated for all to see. This can enable teachers to adapt classroom instruction to more closely fit student needs. Finally, documents support mathematical argumentation in the classroom. A teacher may more easily compare and contrast solution approaches using different representations, for example. Or a teacher might ask students to extend and generalize a previous approach to a mathematical challenge. Again, documents and the classroom network provide a basis for deeper questioning which in turn will lead to deeper learning.

TI-Nspire naturally builds upon the strong graphing calculator research base and aligns with the powerful formative assessment research base. This enables teachers to build upon both NCTM principles and state-level guidelines for the incorporation of graphing technology. The formative assessment capabilities further allows teachers to align instruction with the increasing emphasis on assessment within mathematics education, without reducing emphasis on doing important mathematics. Teachers who want to make a strong research-based case for their instructional plans using TI-Nspire need look no further than the strong and unambiguous research base for graphing calculator use and formative assessment.

Enhanced Representation and Communication of Important Mathematics

TI-Nspire™ math and science learning handhelds feature several striking improvements to both the representation and communication of mathematical ideas. We will address representation first.

TI-Nspire learning handhelds have bigger, sharper screens, allowing graphs to be explicitly labeled and for students to see graphed functions in more detail. In addition, TI-Nspire learning handhelds display expressions in standard mathematical notation and in a textbook-like format (e.g., $\frac{1}{3}x^2$) rather than a computer-like format (e.g., 1/3 x^2). Further, TI-Nspire learning handhelds integrate graphing and geometry in one Cartesian plane, enabling the construction of a line tangent to a graph, for example. Finally, TI-Nspire learning handhelds support showing two or more different representations of the same mathematics on the same screen, with live data flows between them. In combination, the representational features of TI-Nspire learning handhelds provide clearer expression of big mathematical ideas.

These features connect most strongly to the existing research base on linked multiple representations. Researchers have found that students learn concepts more readily when they experience the concepts across different forms of notation (Davis & Maher, 1997; Kaput, 1992; Kaput, Noss, & Hoyles, 2002). A wide variety of multiple representational tools have been developed in mathematics, most of which leverage graphing and geometry as key representations. Small-scale studies with such tools amply demonstrate that a multiple representations approach can produce gains in deep mathematical understanding for diverse children across a range of settings (Roschelle, Pea, Hoadley, Gordin, & Means, 2000). For example, after a pre-algebra course that emphasized multiple representations, anchoring mathematics in a meaningful context, and cooperative problem solving, students were better at representing and solving function word problems than students in a control group (Brenner et al., 1997). A meta-analysis that summarized findings from over 100 research studies involving over 4,000 experimental/control group comparisons found that both (1) representing knowledge graphically and (2) using manipulatives to explore new knowledge and to practice applying it had a large effect size (Marzano, 1998). A multiple representations approach to mathematics combines these techniques and thus should be exceptionally effective.

To illustrate the use of multiple representations, consider the concept of a rate of change, which relates to:

- The value of m in $y=mx+b$
- The differences between sequential rows in a table of x and y values
- The slope of a graphed line

Technology makes it possible to present multiple representations on the same screen and, more importantly, to explore how changing one representation affects the others. For example, a student can increase the value of m and see simultaneously more rapidly increasing table values and a steeper slope in the graph (Confrey & Smith, 1994). The numerical patterns in the linear equation are then visually linked. As the values change, the existence of variable relationship between m and the ordered x and y pairs is immediately seen by students. Such real-time visual modeling enhances interactive feedback to students. Through interactive feedback, students can more rapidly correct misunderstandings (Bloom, 1984). With the support of teacher-guided and collaborative conversation about multiple representations, students came come to understand the meaning of mathematical expressions (Roschelle, 1992). Consequently, researchers in both science and mathematics have identified linked dynamic representations as one of the key benefits of technology (Kaput, 1992; Kozma, Russell, Jones, Marx, & Davis, 1996).

TI-Nspire handhelds also enable communicating with representations via the TI-Navigator classroom networking technology. As mentioned in the formative assessment section, classroom networking can be used to increase feedback both to students and to the teacher. However, TI-Navigator should not be understood narrowly as a testing capability. TI-Navigator also makes possible new forms of classroom communication. With TI-Navigator, teachers can send students one or more screens of preconfigured representations and thus instead of communicating about which buttons to push to get started, teachers can focus their discussion on the mathematics at hand. Further, teachers can harvest examples of work from students and rapidly bring those examples to the front of the classroom for discussion. Even more profoundly, teachers can aggregate student work and engage students in participatory simulations. (These advanced capabilities will be described in more detail below in the "Opportunity to Learn" section).

For example, consider the aforementioned use of multiple representations for the concept of rate of change. By using TI-Nspire with the classroom network, a teacher could begin the lesson by sending students a document that combines the three representations (i.e., equation, graph, and table) on the same screen. Instead of using valuable classroom time to tell students how to set up a screen with all three representations, the teacher can now immediately engage students in exploring the consequences of varying "m" vs. the consequences of varying "b." Further, the teacher can ask students to find a way to make a downwards slope. By harvesting student work, the teacher can engage the classroom in looking at examples and having a discussion around the question "What is the same about the value of 'm' in all the different, downward-sloping lines that you made?"

The representation and communication features of TI-Nspire handhelds fit together to support the teacher in making mathematical thinking and communication the focus of the mathematics classroom. The bigger screen and clearer labeling should support teachers as they introduce and explain the meaning of each representation. For lower-achieving students, in particular, teachers will likely need to reinforce the meanings of each axis of a graph and the basics of plotting and reading points. Because TI-Nspire learning handhelds support more than one representation on the same screen, teachers can now more readily focus attention on how the same mathematical property looks across representations., The availability of geometry in graphs enables teachers to better integrate geometric and algebraic concepts, such as "this rectangle represents the area under this section of curve." The better capabilities to link dynamically across representations should enable teachers to explore the effects of changing variables and to engage students in the context of real world data. In support of these representational capabilities, the classroom networking capabilities can allow teachers to see what students are thinking and doing in order to help them improve. Teachers can allow students to communicate by sending mathematical representations to the shared display and communication, which should enable students to better explain their work and support their arguments. Finally, as described in the slope example above, teachers can build classroom discussions around the similarities and differences in students' work. Through attention to student work, teachers can show that student mathematical ideas and explanations are valued.

Deeper Opportunities to Learn

Using the metaphor of a document to organize classroom work with technology has a long history in educational technology, beginning with Hypercard (Means et al., 1993) and continuing with key innovations, such as the "Progress Portfolio" (Loh et al., 1997), which enabled students to document what they were learning with technology. A document model fits classrooms well because teachers and students are already accustomed to assigning, working on, collecting, annotating, and discussing documents, such as worksheets and homework. Much of the ordinary work of mathematics classrooms is already document-based. Now students can carry their own document-based learning technology in their hand and teachers can integrate the work students do on handhelds into the document-based flow of classroom activity.

In TI-Nspire™ math and science learning handhelds, a document presents a sequence of pages of work that can be saved and later opened. Each page can contain more than one mathematical representation. For example, a teacher can construct a page with a table to the left and a graph to the right and the same function appearing in both. This page may be saved and later re-opened. Using a set of related pages, a teacher can present a sequence of ideas from simple to complex, all of which may be prepared in advance. An obvious advantage of documents is that they can be used to emphasize mathematical structure and can organize an extended sequence of mathematical activities. Without documents, it is difficult to structure activity across more than one screen and teachers spend valuable classroom time describing which keystrokes are necessary to navigate to or produce a particular display.

A classroom network is a natural complement to a document-based approach. In normal classrooms, teachers pass out and collect assignments in the form of documents. When TI-Nspire handhelds are used in conjunction with the TI-Navigator network, teachers will be able to rapidly distribute and collect technology-based documents. Moreover, teachers can do more than was ever practical in a paper based format. For example, with TI-Navigator, a teacher can display multiple students' work on one screen. A teacher can ask students to make functions that are equivalent to $f(x) = 2x$. Students may come up with $f(x)=4x/2$, $f(x)=4x-2x$, and many other possibilities. When collected onto one screen via TI-Navigator, a teacher could display the different functions and how they all result in the same graph, powerfully re-enforcing the meaning of equivalence (Wilensky & Stroup, 1999). Without a network, it would be much more difficult to look at the equivalence across student-constructed functions.

The document capability is a new feature in a handheld mathematics platform. Hence there is no direct research relating document features to mathematics learning gains. Nonetheless there are good reasons to be optimistic. The capabilities of documents align with long-standing findings in the scientific literature on how people learn (Bransford, Brown, & Cocking, 2000). This capability can enable teachers to create a more engaged classroom, with greater motivation and achievement. For example, Blumenfeld (1992) summarized the teacher practices that lead to high engagement and achievement as:

- Providing meaningful opportunities to learn
- Using high-quality instructional techniques, such as offering concrete illustrations and analogies and connecting new concepts to prior knowledge

- Pressing students to think by requiring them to explain and justify
- Supporting students by providing samples they can use, modeling the thinking process, breaking tasks into manageable steps, and encouraging collaboration
- Evaluating students frequently, with an emphasis on comprehension and mastery and allowing opportunities for revision

The classroom network capability has been around for a few years (e.g., it is available for TI's TI-84 graphing calculator line) and thus some research has accumulated. In general, this research emphasizes the opportunities to use networks to transform the degree of student participation in doing mathematics in the classroom and to introduce students complex and conceptually difficult mathematical ideas in new ways (Stroup, Ares & Hurford, 2005; Stroup et al, 2002; Hegedus & Kaput, 2004). Below we discuss four alignments between research and TI-Nspire document and networking capabilities. These alignments offer teachers the occasion to enhance their classroom practice by providing students with more meaningful opportunities to learn.

Academic Learning Time

The importance of time is an enduring issue in educational research (Ellis, 1984; Brophy & Good, 1986). A seminal 1963 article (Carroll, 1963) proposed a model of learning in which the two dominant factors are (1) the time needed for learning and (2) the time actually spent in learning. Clearly, as educators seek higher standards of student achievement, the time needed for learning is soaring. However, merely increasing time allocated to learning a particular topic or subject is not enough. Integral to student achievement is ample "academic learning time" (Fisher & Berliner, 1985). Academic learning time is engaged time during which students succeed at tasks that have an appropriate level of difficulty (Cotton, 1989).

Research has found a strong relationship between increased academic learning time and stronger student achievement (Wang, Haertel, & Walberg, 1993/1994). High-quality academic learning time has been found to be particularly important for low achievers; much of the benefit in interventions appears to be related to engaging low achievers in spending more time doing academic work and experiencing success. For such students, increasing the academic learning time reduces their anxiety and enhances their learning outcomes (Cotton, 1989).

While technology is no guarantee of increased academic learning time, teachers can use technology to enhance time-on-task. The timing decisions of task implementation can play an important role in student academic engagement (Henningsen & Stein, 1997). Obviously teachers will want to select products that start up quickly and almost never break. Beyond these basics, technology should reduce the transition time between learning tasks for the student and accommodate tasks of varying levels of difficulty. The document capability of the TI-Nspire learning handhelds is a promising innovation in both respects. First, with regard to transition time, by providing a document for the students' learning tasks, the teacher can reduce set-up time and enable the student to quickly transition from "turn it on" to "begin solving the mathematical problem." TI-Navigator enables teachers to quickly send documents to all students. Second, with regard to level of difficulty, a teacher can provide a document in which there are related tasks at different levels of difficulty. (In addition, see the discussion of scaffolding below—the teacher can provide more support in the document thus making it easier for students to succeed at a difficult task). TI-Navigator additionally allows the teacher to quickly poll student thinking or harvest student work and to adapt instruction accordingly. For example, by providing students with a document that contains a variety of related tasks, teachers can help students at different levels to engage and succeed, thus implementing differentiated instruction (Kameenui & Carnine, 1998) in a practical and meaningful way.

Scaffolding Mathematical Argumentation

The practice of scaffolding is derived from Russian psychologist Vygotsky and his concept of a zone of proximal development (1978). Vygotsky observed that, under conditions of effective support, a child can engage in a more complex performance than the same child can without support. Eventually, as the complexity is mastered, the student can perform the complete complex skill with less and less support. Importantly, Vygotsky found that social support is a profound form of scaffolding; students often perform advanced skills first with the support of a social group and individually only later. The term "scaffolding" comes from an analogy to the construction of a building. When a building is constructed, first scaffolding is erected to support the project. Later, when the building is complete, the scaffolding can be removed. As a pedagogical principle, scaffolding suggests initially providing a supportive social environment and supportive resources that enable students to succeed and then gradually fading the support as students master increasingly complex skills (Greenfield, 1999).

In mathematics, educators often describe important yet complex skills by contrast to what they are not: meaningless manipulation of symbols and algorithms. Important and complex skills in mathematics include making conjectures, investigating patterns, examining premises, and justifying solutions (Cobb & Bauersfeld, 1995; Lampert, 1990). Practically speaking, teachers can help students in these important skills by making collective argumentation a central classroom activity (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Wood, 1999; Yackel & Cobb, 1996).

Mathematical argumentation is hard for students to do alone. Researchers have found that students can develop this skill if teachers provide the right scaffolding (Henningsen & Stein, 1997). Research has found that the best scaffolding is often a mix of social and technological support (Puntambekar & Hübscher, 2005). Social support is needed to establish classroom norms and discussion processes that establish a safe, productive context for mathematical argumentation (Wood, 1999; Yackel, 2002). Technological support can motivate student interest and provide a means for generating, exploring and organizing claims and evidence (Puntambekar & Hübscher, 2005). Students often rely on examples and demonstrations in creating their initial arguments (Knuth, Choppin, Slaughter, & Sutherland, 2002). Technology can provide ways to structure student activity with complex concepts or data sets and thereby provide a supportive context in which teachers can focus on mathematical argumentation and not just manipulation of symbols and algorithms.

For example, a teacher who wants students to focus on examining and predicting patterns in sequences of numbers can provide a spreadsheet environment that supports both exploring the patterns and trying out different formulas to predict the values in a distant place in the sequence (Niess, 2005). In another case (McClain, Cobb, & Gravemeijer, 2000), a teacher worked with researchers to define focused explorations and investigations that students could do with technology. By engaging in mathematical argument around comparison and contrast of student work, students changed their view of what it means to do statistics and developed deeper understanding of statistical concepts.

The capabilities of the TI-Nspire learning handhelds provides an opportunity to both scaffold and fade support for learning difficult mathematics concepts and skills. For example, international comparison tests like TIMMS show that multistep problem solving is difficult for many students (National Research Council, 1999). A teacher who wants students to master a task that requires multistep problem solving could provide a series of pages in a document, each which supports doing one step. Later, this step-by-step breakdown could be removed, so the student must complete the whole problem alone. Using the classroom network, the teacher also has the possibility of making an initial plan for problem solution together as a class and then sending students that initial plan as a document. Students may then work independently or in groups. At an appropriate point, teachers may collect students progress via the network and discuss it with students at the front of the classroom. Thus the network allows working a complex problem through a mixture of teacher-led work and small-group or student-led work. TI-Nspire documents provide a new way to structure materials to scaffold student learning and TI-Nspire networking enables new ways to structure social participation in the classroom to scaffold student learning.

Participatory Activities

Every teacher has the goal of engaging his or her students. “Engagement,” however, like “motivation,” is often thought of as an ingredient needed to support the hard work of learning mathematics. TI-Nspire’s networking features provide a new way to realize engagement and motivation – a way to accomplish these with less reliance on rewards or gimmicks. Using TI-Nspire’s networking capabilities, students have an opportunity to learn by participating in social mathematical activities. Researchers are finding that when social participation becomes more deeply tied to mathematics learning, motivation and engagement come naturally. Students learn more when they are engaged and disciplinary problems and drop out go down (Fin, 1989; Klem & Connell, 2004; Newmann, 1992; Roderick & Engle, 2001; Willingham, Pollack, & Lewis, 2002)."

In participatory activities, social and mathematical aspects of learning are interconnected (Stroup et al., 2002). This makes mathematical learning more akin to play than work—indeed, real mathematicians and scientists tend to describe their peak breakthroughs as arising through playful activities, not through endless computational drudgery (Stefik & Stefix, 2004). Further, research has consistently found that students learn more when encouraged to engage in helping behaviors with their peers, for example when they building on each others’ mathematical ideas, offer constructive critiques, and take complementary roles in a shared project (Webb & Palinscar, 1996). In general, research in the learning sciences has increasingly come to see learning not merely as cognitive, but also as importantly social and participatory (Sawyer , 2006). Participation deeply links to engagement and motivation through the notion of *belonging* (Lave & Wenger, 1991). Students want to learn in order to belong, not only in order to know. When students have a sense of belonging, as well as other components of a healthy climate such as trust and safety, they learn more (Lee et al, 1999). Participatory activities link the opportunity to belong to a classroom activity to the opportunity to learn.

Participatory simulations provide a classic, although not the only form of playful, belonging-oriented activity that leads to important mathematics learning (Stroup, Ares & Hurford, 2005). In a simple participatory simulation, every student controls a point that is graphed on a Cartesian plane. The points are initially randomly distributed all over the plane and students are allowed to play with moving their point randomly, so as to discover how they can control it. Then the teacher asks the students to "make your y-coordinate twice your x-coordinate." As the students move their points, a line suddenly emerges. This line is made only when every student participates and is a shared construction which belongs to the classroom as a whole. The surprising emergence of the line also provides the teacher with a profound opportunity to ask students "why did you get a line?" or "what other rules might make a line?" and thus inquire into the kind of rules that lead to linear functions. In the play a part in the participatory simulation, students are naturally motivated and engaged but also deeply involved in understanding the topic of linear function. Further the students are not just finding solutions, they are involved as a group in generating interesting mathematics (Stroup, Ares & Hurford, 2005).

Another form of social, mathematical activity that draws intense student participation might be termed "participatory narratives." In a language arts class, students might make a story by each contributing a sentence to an evolving plot. Naturally, students are more interested in the resulting story when they each contribute a line than when only the teacher or only a single student are involved. Researchers in the SimCalc project have discovered a powerful analogy in the mathematics classroom: students can each contribute a mathematical function to a group performance that is driven mathematically (Hegedus & Kaput, 2004). Using the SimCalc software, which connects individual student devices via the TI-Navigator network, each student can write a function that will drive the motion of a character. For example, the position functions f(t)=t and f(t)=2t drive a slower and faster animated motion, respectively. Using this capability, students write functions that tell a story together, e.g., "we started at different places, one of us walking slowly and the other quickly, and met at the end of 10 seconds of walking." Participating in performing this story together, via the network, both requires serious mathematical thinking and encourages productive classroom discussion. Indeed, researchers are finding that when students make a mathematical object and contribute it to a group construction on the teachers' display, students can become intensely involved in arguing about the correct understanding of the resulting mathematical phenomena (Hegedus & Kaput, 2004).

Using TI-Nspire, teachers will have opportunities to increase student participation in these and other ways. For example, teachers can encourage students to collaborate on a document that they use to present their shared solution to a mathematical investigation. Further, a teacher can choose one example of student work anonymously and display it to the classroom. Rather than merely critiquing the work, the teacher can ask how it might be improved or extended. Then that students' work could be instantly distributed to all students in the class, who are asked to make it better or extend it further. In beginning a unit, teachers can ask students to individual explore a model that was provided to them in a document. By displaying and discussing what students discover, however partial, incomplete, or uncertain, the teacher can set the stage for the important mathematical learning that will be addressed as the unit progresses.

Reflection and Revision

The landmark volume entitled "How People Learn" (Bransford, Brown, & Cocking, 2000, p. xii) summarizes the results of decades of learning science research and highlights the discovery that:

> Individuals can be taught to regulate their behaviors, and these regulatory activities enable self-monitoring and executive control of one's performance. The activities

> include such strategies as predicting outcomes, planning ahead, apportioning one's time, explaining to one's self in order to improve understanding, noting failures to comprehend, and activating background knowledge.

The research basis linking self-regulation and achievement is exceptionally strong (e.g., Zimmerman & Pons, 1986). For example, students who review a prepared problem solution and "self-explain" the steps of the solution learn more than students who do not study this way (Chi, deLeeuw, Chiu, & Lavancher, 1994). When students learn these self-regulatory, meta-cognitive skills they become better learners. This insight reinforces a dual mission that teachers have always understood: mathematics teaching should both result in mastery of topics and also prepare students for future learning. Yet just as "Rome was not built in a day," a busy teacher cannot expect to transform students' meta-cognition while also addressing a jam-packed curriculum. Two key activities that teachers can sensibly focus on are reflection and revision, both of which relate to essential mathematical practices, such as abstraction, generalization, proof, and communication.

Reflection, that is engaging in the active processes of evaluation and justification, is a necessary component of key mathematical practices, such as conjecturing, generalizing, abstracting, or critiquing, and is central to deep learning of mathematics (Wheatley, 1992). Education theorists have long supported the entwined processes of reflecting and learning. Polya (1957) proposed reflection as one of four key mathematical processes. Dewey (1933) identified reflection as the core activity of advanced thinking, leading to the concept of experts as "reflective practitioners" (Schön, 1983). Freudenthal (1973) suggested that the essence of reflection in mathematics is shifting one's viewpoint to gain additional insight (also see Mason, 1989). More recently, reflective activities, such as classroom discussion and elaborate student explanation, as guided by the teacher are recognized promising practices (Grouws & Cebulla, 2000; The Access Center, 2006). As students reason their answers with others, they can be prompted to reflect on their practice. With regard to mathematics specifically, research closely grounded in classroom realities has noted that reflection involves specifically talking about mathematical activities, not just mathematical results (Cobb, Boufi, McClain, & Whitenack, 1997). To reflect on mathematical activities, those activities need to be captured and displayed in a visible form. A document seems ideal as a way to capture mathematical activity as the basis for reflection.

Revision is also inherent to the mathematical processes of abstraction, generalization, and justification. In research on how students can most effectively learn math, revision has gradually moved from an optional "extra" to being a core instructional element (Carpenter & Romberg, 2004). Revision is not just about improving the final product, it also provides exceptional opportunities for students to learn and teachers to teach (Fitzgerald, 1987). Mathematicians spend an exceptional amount of time reflecting upon and revising their proofs, whereas in school most students solve a problem and then move on to the next problem. We see documents as opening the opportunity for teachers to engage students in revision: making their mathematical arguments stronger, their insights clearer, and their generalizations broader.

Teachers can use the document capability of the TI-Nspire learning handhelds to engage students in reflection and revision, thus helping students to learn how to learn. A teacher can structure a document to have an early page that prompts students for predictions (e.g., what will happen to the solution to simultaneous linear equations as the slope in one equation is increased?); students can later compare their prediction to what they learned by exploring a page that graphs two linear equations. A teacher can provide worked examples and ask students to explain these to themselves in preparation for solving a related problem. In turn, a teacher can engage students in discussing the problem solving process. Furthermore, documents can support the important mathematical practice of returning to prior work after a new insight and revising the mathematics to make it better.

Conclusion

Teachers can draw upon three levels of research alignment as they plan classroom use of TI-Nspire™ math and science learning handhelds to enhance student achievement. First, strong summative scientifically-based research supports the effectiveness of graphing calculators and formative assessment in enhancing student achievement. Second, long traditions of mathematics education research demonstrate the benefits of multiple representations, especially when combined with activities that engage students in doing and communicating important mathematics. TI-Nspire handhelds provide linked multiple representations and engage students in doing and communicating mathematics via the TI-Navigator classroom network. Third, TI-Nspire aligns with what scientists know about how people learn. Using the new document and network capabilities, teachers should be able to increase academic learning time, scaffold mathematical argumentation, increase student participation and encourage reflection and revision.

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