A comprehensive framework for improving mathematics in low-performing secondary schools

Research Note 16

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The challenges of teaching mathematics are numerous even when students are high performing. When students are performing poorly (and perhaps are part of a population that is associated with poor mathematics abilities), the challenges increase significantly and can become daunting. As educators face choices regarding how to help, organizing an effective improvement strategy can be difficult because so many factors affect student learning.

To improve mathematics in low-performing schools, educators should address a broad range of factors systemically, including an intensification strategy, coherent curriculum, effective pedagogy, deeper teacher mathematics knowledge, positive social factors and supportive organizational structures.

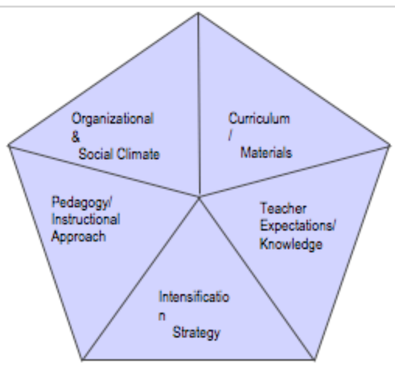
This note summarizes a research-based Conceptual Framework developed to help educators and researchers comprehensively understand the factors involved in improving mathematics instruction. They can then make better choices about which intervention to pick, or how to supplement or customize a chosen intervention for their local setting.

To develop the conceptual framework, a team of researchers examined two research literatures: the cognitive sciences research literature on how people learn and think about mathematics, and the international comparison literature examining approaches to teaching and learning mathematics in different countries. In addition, the framework was informed by a review of 17 interventions in mathematics.

The team that created the framework included mathematics, evaluation, and reform experts from SRI International, and leading experts in mathematics education from universities and the National Council for Teachers of Mathematics (NCTM). Funding for the project came from the National Science Foundation (NSF).

The conceptual framework highlights a critical insight related to the education of students entering high school one or more grade levels behind in mathematics. For these students there is no single "silver bullet" which will close the gap.

An effective strategy must address the mathematics curriculum and instruction, as well as motivational and social factors. The research review concluded that an effective strategy must include five dimensions of students' school experiences, as shown in Figure 1, below:



- Intensification strategy - the way in which students are given more content and or more time to learn within a given year.
- Curriculum - what students are expected to learn, and its coherence.
- Pedagogy/ instructional approach - the kind of instruction given to students.
- Teacher expectations/ knowledge - teachers' attitudes regarding their students' capabilities, as well as their knowledge of mathematics and how to teach it.
- Organizational/social climate - factors such as the way students are grouped for mathematics instruction and support structures provided for learning.

Figure 1: Conceptual Model: Five Dimensions of an Intervention

The description of each dimension below contains insights from the research and best practices. These descriptions may point developers and educators to practices or new methods that could potentially increase student achievement.

1. Intensification Strategy

Because these students are behind, they need to follow a more aggressive schedule to catch up. The research-based principle of “academic time on task” holds that an increase in time spent on a subject results in an increase in learning outcomes, but does not specify any particular intensification approach.

Some intensification interventions try to increase amount of time on mathematics tasks by increasing class length, or by having students take two mathematics courses simultaneously. After school programs are another way to increase time on task, but if the content between the in-school mathematics course and the after-school program is not aligned, it can cause confusion and harm struggling students.

Another approach is content acceleration. Content acceleration can refer to several strategies designed to help students move ahead at a more rapid pace than normal. In most U.S. math classrooms there is so much review of basic information that it is difficult to get to new material (Stevenson & Stigler, 1992; Schmidt et al., 2005).

For younger students, one can accelerate content by introducing advanced topics early in middle or elementary school. However, when students are already in high school, one strategy is to turn a one-year course into a one-semester course with longer course meetings. Other intensification strategies are discussed in the full research report (a Web link to the report is provided at the end of this research note).

2. Curriculum

Curriculum is considered integral to any attempt to improve mathematics instruction. One often-discussed theme is curricular coherence. Much of the research on curricular coherence comes from the international comparative research examining curriculum, teaching and student performance.

Schmidt and his colleagues (2005) found that in the United States, most of the curriculum covers an enormous number of topics and lacks a core set of ideas linking concepts in the curriculum. Consequently, American curriculum has been called “a mile wide and an inch deep” (Schmidt et al., 2005). The large number of topics and lack of coherence in U.S. curriculum usually means the topics are *not* presented in a meaningful way (Hiebert and Carpenter, 1992).

By contrast, in countries that perform well on mathematics assessments (like the Trends in International Mathematics and Science Studies, TIMSS), there is often a standard, well-organized curriculum that uses core concepts as a way to help students anchor their new knowledge, and mathematics is presented as a coherent set of ideas related in logical ways (Charles, 2005).

Schmidt and colleagues consider the disorganization in curriculum a primary reason for U.S. students’ poor performance on international mathematics assessments and the vast achievement gap within the U.S. between high-poverty students and more affluent students. NCTM created *Curriculum Focal Points* to cluster concepts for grades K through 8 (NCTM, 2006), and is developing a similar resource for high school mathematics.

Other commonly discussed issues involved in curriculum discussion include the importance of both procedural and conceptual fluency in mathematics, teaching meta-cognitive strategies to help students perform better, introducing multiple representations such as graphs, tables, symbolic expressions and narrative descriptions at an earlier point in education to help students gain a more complete understanding of the concept, and how to use the different representations to reason better (Goldin, 2000; Miller, 2005).

Technology can be a powerful means to focus students on the meaning of mathematical representations and to develop conceptual understanding and mathematical reasoning alongside the development of procedural knowledge (Kaput, Lesh & Hegedus, 2007; Roschelle et al, 2007).

3. Pedagogy and Instructional Approach

Research from the international comparative literature highlights pedagogical differences that seem to impact students' mathematics learning. For example, teachers in Asian countries tend to assign students to do richer mathematical tasks. In Japan, lessons follow a structure where (a) students try something out; (b) a teacher provides direct instruction on that challenge; (c) the students work on the problem again (often as a whole class); and (d) the lesson ends with a review of the concepts covered (Siegel, 2004).

In this structure, Asian teachers have the opportunity to explicitly point students to critical concepts, use student errors to better explain math reasoning (Stevenson and Stigler, 1992), and require students to grapple with concepts, and articulate mathematical ideas (NCES, 2000).

By contrast, the way lessons are presented in the U.S. offers less rich mathematical experiences. For example, students are not given time to think or articulate about mathematics, since teachers typically ask "yes/no" questions and emphasize procedural accuracy (e.g., the "right way" to solve a problem). Because the lessons offer students limited responsibilities, they do not get the opportunity to reason through the concepts themselves (Stevenson and Stigler, 1992).

Related to the importance of thinking and reasoning about mathematics is making student thinking visible, a critical component of the learning process. When students explain their thinking they are required to organize thoughts clearly in order to communicate them to others (Chapin et al. 2003).

In a synthesis of effective instructional strategies, Baker and his colleagues recommended that "segments of mathematics instruction should target teaching students to generate explanations of math concepts in their own words and to justify the methods they use to solve problems" (Baker et al., 2002).

Using instructional approaches that emphasize group work can help support learning in mathematics better than traditional instructional methods (Slavin, 1990; Marzano et al., 2001). Benefits of cooperative learning include increased knowledge and skills, increased conceptual understanding, improved attitudes or motivation, improved communication skills, and improved social skills (Davidson, 1990). Using networking technology in classrooms can create new and enhanced opportunities for students to work on mathematics collectively, leading to increased learning (Roschelle et al. 2009; Stroup et al. 2002).

Different pedagogical approaches can also support the development of new cognitive skills necessary for the transition from simple arithmetic thinking to the more abstract thinking required by algebra (Johanning, 2004).

For example, when students have problems with negative, irrational, and imaginary numbers, Sfard (1995) recommends a pedagogical approach that allows students to work without understanding, just completing the calculations and techniques to slowly and iteratively construct their own understanding of the abstract objects.

This approach requires patience on the part of the teacher and the student, but a complete conceptual understanding cannot be obtained without experience. In other cases, technology-based representations can directly link graphs and equations to narratives and animations, providing another way for students to work with the concepts and connect mathematical abstractions to familiar situations (Roschelle, Kaput & Stroup, 2000).

Formative assessment is another key pedagogical strategy to scaffold understanding and correct misconceptions more quickly. Research has shown that formative assessment helps reduce the learning gap between struggling and positive achieving students and raises overall achievement levels (Black & William, 1998). Using formative assessment as an instructional practice can be challenging for teachers.

Teachers need to learn to create assessments that align with learning goals and activities; then they have to interpret the results of the assessments; and finally they must be able to make changes to instruction based on those results in a timely fashion to affect student learning.

In a recent large-scale scientific study, classroom network technology has been shown to be effective in increasing students' Algebra scores by increasing formative assessment (Owens et al, 2008; see also TI Research Note #14). When a teacher has more frequent and accurate information about students' mathematical thinking, they can often adjust instruction to increase learning.

4. Teacher Knowledge and Expectations

Research suggests that greater teacher content knowledge is a contributing factor to increased student learning (Ball, Hill and Bass, 2005). Teachers in Asian countries have stronger mathematics knowledge and more training, on average, than teachers in the United States, and students in Asian countries typically perform better than students in the U.S. on International Math Assessments (Ball, 2003; Siegel, 2004; Stevenson, 1998).

In addition to content knowledge, a teacher also needs pedagogical content knowledge, or specialized mathematics knowledge for teaching that includes representing the subject to make it understandable to others; knowing what topics will be easy or not; and insight about what preconceptions students of different ages and backgrounds have (Shulman, 1986).

A successful mathematics teacher needs a deep understanding of mathematics content to create mathematically rich task structures and to adapt to meet the needs of individual students (Ma, 1999); professional development focused on increasing teacher knowledge of mathematics may increase student performance.

Another factor that affects student learning outcomes is teacher expectations. Teachers with low expectations of their students will most likely only teach remedial skills and not move on to teach more advanced concepts or stress conceptual understanding (Romberg, 1984).

Teachers too often believe that attempting to teach higher-order thinking is inappropriate when students are struggling (Zohar, et al., 2001). Only if teachers consistently set high expectations by engaging all students in challenging mathematics can students be expected to learn more challenging mathematics.

Many school districts have utilized professional development efforts to raise teacher expectations of how low-income African American and Latino students can perform in mathematics. One method of raising teacher expectations is to involve teachers in investigations of student work, when students are given more challenging assignments and supported to do them.

5. Organizational and Social Climate

The organizational structure and social climate of the school can have a profound effect on students' performance. Two contrasting beliefs are often found. Some parents, teachers and students believe that everyone can develop their ability to learn mathematics. Others believe that mathematics is an innate skill or trait – that some people are "good at mathematics" and other people are not.

Asian countries tend to subscribe to the former belief, whereas in the United States, parents and teachers are often willing to accept that a student is innately "not good at math." Consequently, students labeled as not good often do not try, since they feel their efforts will be futile (Stevenson & Stigler, 1992).

A simple, but effective intervention taught students that the brain was like "a muscle" and if they exercised it in mathematics, it would get stronger. U.S. students who were told this analogy did better than students who were not told (Blackwell et al., 2007).

Other negative effects on motivation and expectations can occur through "stereotype threats" which occur when a group that is often stereotyped as poor in mathematics such as female, African American, or Hispanic, is reminded of this stereotype (Cohen, et al., 2006). Spillane (2002) found that many teachers have a "deficiency" view of their "disadvantaged" students. Whether the negative belief systems are about race or gender, teachers need to be challenged, typically through professional development.

Teachers need to learn how to best support students that are not part of the "typically good at math" group; as teachers learn how to support students and raise their own expectations, they can help every student learn mathematics. One key step in doing this is for administrators to raise leadership in mathematics learning up alongside more traditional issues such as safe and drug-free schools.

Parental beliefs and home support for mathematics performance also relate to student success (Cooper & Robinson, 1991), so outreach efforts to help parents understand what they do at home is important for mathematics learning as well.

Utilizing the Conceptual Framework

Seventeen available mathematics interventions were examined with the framework; it was found that many stressed curriculum or pedagogy, or social or motivational factors, but none comprehensively integrated all five of the areas identified by the conceptual framework to be important to struggling students.

Because of this, struggling students are not getting all the types of support they need. In interviews, the designers of the interventions expressed that they knew many different issues were important to the success of an intervention, but they assumed that other aspects were already in place or could be taken off the shelf (e.g., adopting a textbook).

For educators, the comprehensive conceptual framework can provide an understanding of what the intervention lacks so that they can be aware and work to give appropriate additional support to their students.

The framework takes a systemic perspective on the problem—a comprehensive perspective that is especially needed in low-performing schools. Educators who are designing an approach to help students in their school may find the framework and the descriptions of the components helpful.

The framework identifies the elements of the system that need to be supported to help low-achieving students succeed in algebra. The strategies of best practice that are suggested by the framework can potentially raise student achievement, if they are all used together in a coherent way.

For more details on the framework, the project's complete findings are available, along with descriptions of the 17 interventions and summaries of key articles in the literature, on the project's wiki at:

https://wiki.sri.com:1800/display/REESEMATH2/Welcome+Letter and use the username *visitinguser* and the case-sensitive password *SRI_Wiki*09*

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