Real World Applications on Student Understanding in an AP Calculus Class: An Action Research Project

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Introduction

With the recent advent of handheld graphing technology and its increasing capabilities, high school mathematics teachers have been presented with an interesting dilemma: to what extent should teachers use graphing display calculators (GDCs) to maximize student understanding? In recent years researchers have been encouraging their use, particularly in higher level mathematics (Ellington, 2003; Kaput, 2007; NCTM, 2000, van Streun et al, 2000). The multiple representational views of certain mathematical topics such as functions allow the student a more dynamic approach to learning mathematics. Students are able to manipulate graphs, equations, and expressions that allow for a more student-centered, kinetic approach to learning. Additionally, there are many functions that allow for a more efficient and exact solution to be found. However, because the technology is relatively new, there are still many unanswered questions related to graphing technology and student understanding. Many teachers are resistant to the technology for fear it is replacing student understanding rather than enhancing it. Others simply question the role GDCs should play in mathematics curriculum as some features replace skills that traditionally took weeks to master by hand. Nelson, Palonsky, and McCarthy summarized these concerns citing a need for “reasoned criteria, solid evidence, and critical skepticism to make adequate judgments about the relative value of technologies,” (2007). To these ends, this action research study will combine the normal discourse of a curricular unit on

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the applications of the derivative in an AP Calculus course with the use of GDCs through real world applications to determine the extent it will impact student understanding.

Focus Question

To what extent will the integration of real world applications through the use of graphing technology impact student understanding of AP Calculus material for high school students?

Setting

This research project takes place at a high school in a small blue collar community. The school is somewhat diverse (63% Caucasian, 20% Hispanic, 17% Black) and caters to a large economically challenged population (52% free and reduced lunch). The school is divided into several vocationally-based academies including among others automotive, nursing, and business. Students in the AP Calculus class are mostly from the Honors academy; however, one student is in the Digital Design academy. The class is relatively small with fifteen twelfth grade students of which twelve are non-Hispanic Caucasian, two are Hispanic, and one student is multi-racial. There are ten male and five female students. The physical arrangement of the classroom is student centered allowing for students to cluster themselves in small groups and the class meets four times a week for 60 minutes each session. While most students bring their own GDC, there are class sets of multiple versions for student use (most students use the TI-89 or TI Nspire CAS).

Literature Review

Richard Shelly, while writing on the Roanoke Valley Governors School, a school at the forefront of cutting edge, technology-based school design, noted that “instructional technologies are most effective when they are used to enhance and supplement the curriculum,” (2002). While this quote was written in justification for their methods of mathematics classroom design, the applications transcend the physical layout of the classroom. Teachers are finding escalating uses in mathematics for the increasing functionality and capability for complex modeling of GDCs. This is particularly true in higher level mathematics as the curriculum matches these increased functionalities. Haas confirmed this through a meta-analysis of 35 experimental studies that found “Technology-aided instruction showed its strongest effect size for advanced-algebra course students,” (2005). However, a student cannot rely on a calculator to teach the curriculum. Instead, the instructor must use the GDC to enhance their lessons to further student understanding.

Donna Hubbard in an action research project confirmed this by writing, “the objective of this project…to apply graphing calculator skills when solving mathematical problems…was accomplished through increased instructional emphasis on the functions and capabilities of the graphing calculator,” (1998). However, knowledge of the functions of a calculator does not automatically equate to increased student understanding. Rather, instructors need to know *how* and *why* students use those functions and adapt their curriculum to meet those needs. Doerr and Zangor recognized this after performing a qualitative study on two pre-calculus classes. They report on “five categories of patterns and modes of calculator use by the students,” (2000). These include the use of GDCs as a computational tool, transformation tool, data collection and analysis tool, visualizing tool, and a checking tool, (Doerr and Zangor, 2000). It is imperative for teachers to recognize these varying tools and teach students how to use them accordingly. Researchers have found the multiple representations a GDC provides are the primary avenue through which students use each of these tools. In examining a 10th grade trigonometry student, Sang Sook Choi-Koh noted “free movement [between multiple representations] that was accomplished by pushing a button between the graphs and algebraic functions, between algebraic functions, and between the graphs facilitated his experience of the dynamic visualization of trigonometric properties," (2003). Piez and Voxman furthered this thought stating, "Because each representation emphasizes and suppresses various aspects of a concept, we believe that students gain a more thorough understanding of a function if it is explored using numerical, graphical, and analytical methods," (1997). With students of varying learning styles, instructors will find students individualize how they use GDCs to further their understanding. This flexibility offers a student more tools to find success. Piez and Voxman, through their work with Algebra II students, found that students "need to be strongly encouraged, possibly required, to work with multiple representations; activities such as these give students that experience and help them develop flexibility in their problem-solving abilities. In the long run, students who have more flexibility will be more successful in solving a wide range of problems," (1997). To help students understand and effectively utilize this flexibility, Sang Sook Choi-Koh emphasizes the importance of posing appropriate questions so that students can utilize the functions of graphing calculators in answering. "Students should not use the calculator just to get a quick answer to a question by pushing a button. Instead, teachers should ask "why" and "how" when trying to stimulate students to think mathematically and meaningfully," (2003).

Methodology

This study took place over the course of four weeks. The first two weeks involved a traditional discourse on the applications of the derivative including the increasing and decreasing nature of functions, concavity, and local and global extrema. Lessons were delivered through PowerPoint presentations and involved note taking, teacher directed examples, student led examples, and a follow up assignments completed partially in class and partially at home.

Additionally, daily formative assessments were given through the use of non-calculator AP multiple choice questions from prior years' assessments. Graphing calculators were allowed throughout the unit but were never stressed by the instructor. Upon completion of these lessons, a unit test was given as a summative assessment on student understanding. Then, to determine the effects the integration of real world applications through the use of graphing technology will have on student understanding, students completed a mini unit including three real world application labs focusing on the use of graphing calculators. Each lab was completed in small groups to encourage discussion and recapped as a large group to synthesize the lesson. After this mini-unit, students completed a final summative assessment to be compared with the first assessment. In order to insure test integrity, questions on both assessments were taken from previous administrations of the AP Calculus AB exam. Students then completed the unit with an anonymous, online survey to garner specific, personal feedback on the impacts of the real world, GDC-driven unit.

Results

To assess the effectiveness of the use of GDCs though real world examples I designed a pre- and post- test to be given before and after a mini-unit involving three real world labs. The pre-test was administered after the normal discourse of the curricular unit on the applications of the derivative. However, shortly after administering the assessment, I had doubts regarding the validity of the scores. Because of the nature of the study and not wanting to penalize students unnecessarily, I did not factor the grade of this assessment into their nine weeks grade. Because of this, students did not prepare themselves as well as they would have had this test counted. Regardless, increases in scores from the pre-test to the post-test should not be summarily dismissed as the increases in scores were not totally due to a lack of preparation and an increased time in studying the content. In fact, an examination of the test scores found in Table 1 shows the class average increasing 34.5 percentage points. Further, not one student decreased scores from the pre-test to the post-test.

Table 1: Pre- and Post- Test Results

	Pre-Test		**Post-Test**	
	Points out of 50	Percentage	Points out of 50	Percentage
Student 1	18	36	39.5	79
Student 2	22	44	45.5	91
Student 3	30	60	48	96
Student 4	24	48	46	92
Student 5	21	42	38	76
Student 6	10	20	28	56
Student 7	21	42	34	68
Student 8	38	76	49	98
Student 9	23	46	40.5	81
Student 10	24	48	45	90
Student 11	25	50	40.5	81
Average	23.3	46.5	40.5	81

Researchers have pointed out an obvious fact regarding math education and the use of GDCs, that is, it is extremely difficult to see how a student uses a calculator and for what purpose (Williams, 1993; Berry, Graham, & Smith, 2006). While keystroke recorders have been used in subsequent studies to examine the exact sequence of keystrokes students use in solving problems, this software was beyond the resources this action research study provided. Instead, an anonymous online survey was developed to garner feedback from students as to the effectiveness of the use of graphing technology through real world examples.

The most immediate result from examining the surveys came from the overall understanding of the material before and after the unit on GDCs. 50% of the students surveyed responded that they understood little of the content assessed before the mini unit on GDCs.

However, after the mini unit, 100% of those surveyed said they understood at least a good amount of the material with 38% stating they felt very comfortable with the content assessed.

One of the most surprising results from this survey ran counter to what some researchers considered a limitation of GDCs. In Doerr and Zangor's qualitative study on the use of GDCs in a math classroom, they summarized these limitations by stating "the graphing calculator emerged as a constraint and limitation in two ways: (1) students' attempted uses of the device as a 'black box' without attending to meaningful interpretations of the problem situation; and (2) the personal (or private) use of the tool," (2000). However, student responses from the survey showed that relying on the calculator to do complex mathematical operations allowed them to focus on the overall problem at hand. For example, one student responded, "The graphing calculator didn't increase my understanding on the applications of derivatives, but it made it easier to do the problems." Another student stated "being able to use a calculator to reduce human errors" contributed to a greater understanding of the material. Finally, one student stated "the calculator can easily take care of the tedious algebraic simplifying that is so prone to mistake by hand. The use of the calculator boosted my confidence on the second test date, which helped me a lot." While students did not address in the survey the personal use of the GDC, through informal observations and discussions, students expressed the use of the GDC actually increased the group learning dynamic leading to increased individual understanding. When one member of the group did not get the same answer as the others, it forced the group to examine the key strokes to locate the error. This error analysis performed as a group led to a greater student understanding of the material.

The benefit to student understanding of the multi-representational function of GDCs previously cited in this paper was also confirmed through the survey. One student summarized this thought stating, “I feel since I'm more of a visual type of person, at least when it comes to math, that the ability to see the graphs I’m working on helped substantially.” Further informal discussion in class further confirmed this sentiment. Students cited several “Aha” moments when looking at graphs, tables, and algebraic results of functions and their derivatives.

The survey did reveal one limitation in that the continued study of the content also led to a greater understanding during the post-test. A student summarized this thought by stating in the survey, “The extra time helped the material sink in better.” However, this limitation was tempered by the vast amount of comments touting the benefits of the GDC unit. Additionally, of the 8 students who took the survey, 2 stated they mostly agree and 4 stated they totally agree with the statement “The use of graphing calculators greatly contributed to my understanding applications of the derivative.” While there is truth in the thought continued study will result in higher scores, there is a point of diminishing returns where continued study utilizing the same methods will not return the same rate of improvement. That is why it is incumbent on teachers to integrate the use of graphing technology throughout their curriculum.

Conclusion

It is not uncommon for AP courses to be taught with one goal in mind: for students to pass the end of course AP exam. While this should not be accomplished at all costs where the ends justify the means, there is considerable debate as to the extent the AP exam should play in curricular decisions. For example, in AP Calculus AB, should the topic of integration by parts be taught? This topic is not assessed by the AP Calculus AB exam but there is validity in the argument that it is a logical extension to the instruction of integration and would naturally be found in any college level calculus class. Regardless of which side of the debate one ends up on, an AP instructor cannot ignore the 800 pound gorilla that shows up at the end of the year. To be more effective, instructors need to examine fully all aspects of the curriculum and how it is delivered and assessed ensuring they are done in the most efficient and effective manner resulting in positive student growth. This research study set out to examine the impacts of graphing calculators on student understanding in an AP Calculus class and found them to be strong enough to warrant their increased use in subsequent years. In fact, parallels from this study can be made to all secondary mathematical subjects. While several positive impacts were identified, still more research is needed for classroom teachers to fully understand how to integrate this technology to better student understanding.

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