

TI-Nspire™ CAS Learning Technology Helps New York Inner City ‘Math A’ Educator Build Deeper Conceptual Understanding Among Students

Case Study 5

Teacher/Researcher – Eric Butterbaugh, Bread and Roses Integrated Arts High School, New York, NY



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Teacher/Researcher	Eric Butterbaugh
Location	Bread and Roses Integrated Arts High School, New York, NY
Course	Math A
Grade	9 and 10
Student Profile	28 students each
Technology	TI-Nspire CAS technology with computer software, calculators, projector/whiteboard

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Setting: Located in New York City’s central Harlem, Bread and Roses Integrated Arts High School serves a culturally diverse student population of 500 and a faculty of 45. The school features an academically rigorous curriculum with integrated arts provided by resident artists. In 2005, 76% of students qualified for free and reduced lunch. At that time, the student body comprised 52% African-American, 46% Hispanic, 1% Caucasian, and 1% Asian and others. Mr. Butterbaugh’s 2006-2007 9th Grade Math A class meets daily as part of a 100-minute block schedule. Mr. Butterbaugh’s 2006-2007 10th grade class meets daily for a single 48-minute period. Each class has 28 students.

Curriculum & Teaching: Mr. Butterbaugh uses the 2004 textbook edition of Allan Bellman, et al, New York Math A: Single Volume Version, published by Pearson Prentice Hall. He supplements the textbook with old Math A Regents test questions, which he uses for class work, homework and tests/quizzes. He uses computer software for specific types of class work, homework and exams.

Students generally work in pairs, and in most cases, he pairs one higher-performing student with a lower-performing one.

To assess students formatively, Mr. Butterbaugh uses a combination of class work, student questions/responses, student discussion/collaboration in paired work, individual journal entries and writing prompts, quiz documents, a problem collected at the end of the class, homework, and biweekly quizzes of 5-7 problems. Summative assessments are composed entirely of old Math A Regents problems he selects, and usually consist of about 20 multiple choice questions and 4-6 free response questions.

In the course of the school year, he estimates that 80-85% of the planned curriculum was covered.

Mr. Butterbaugh used both the TI-Nspire CAS handheld, and the complementary laptop-based software. He points out that in his school, laptops require advance arrangements to schedule and distribute while learning handhelds are always available to students.

Results: Mr. Butterbaugh reports that the most immediate advantage of using TI-Nspire™ CAS learning technology is its potential to promote inquiry-based learning. He comments:

The most exciting part of my experience with the TI-Nspire CAS technology is that **it has redefined my ideas about the types of concepts that can be enhanced through inquiry-based instruction.** This is particularly evident with CAS (Computer Algebra System), as it can be used to promote the investigation of strictly algebraic concepts. For instance, in the past I have taught students how to simplify exponential expressions by showing them one or two examples, explaining the process, and asking them to replicate this process with additional problems. With TI-Nspire CAS technology, however, students can simplify examples using the handheld, identify the patterns, make predictions, evaluate their predictions, and ultimately create their own problem-solving process and apply it to future situations. This is just one of countless examples of how the TINspire technology can be used to turn a traditional “chalk-and-talk” lesson into a student-centered investigation. It is important to continuously identify and reflect upon the core concepts of a curriculum, and I think that future TI-Nspire users will benefit from exposure to and discussion about the types of broad mathematical concepts that can be enhanced through inquiry-based instruction.

He continues:

Often math teachers are encouraged to cultivate deeper conceptual understanding among students. Most teachers, however, have not experienced this as students or observed what it looks like in a classroom. Neither did I until I started teaching with the TI-Nspire technology. TI-Nspire technology offers a dynamic environment which emphasizes the connections between multiple representations of a given concept. As a result, instructors are able to focus on the quality of instruction rather than the quantity of instruction. Therefore, my main recommendation would be to envision a classroom where each new concept is presented to – or discovered by – students using at least three different representations or methods. Imagine the types of questions students will ask when they can solve a problem three different ways. I would also want my colleagues to get TI-Nspire technology into their hands as soon as possible.

The ability of TI-Nspire CAS technology to provide multiple, dynamically linked representations of graphs, equations and tables proved particularly useful in teaching graphing and linear functions, one of the most important concepts in the 9th grade Math A curriculum. Mr. Butterbaugh explains:

The dynamically linked graphs and equations allowed students to investigate a line with the translational and rotational cursors and clearly see the connection between the slope and y-intercept of a graph and the variable and constant terms of an equation. Adding a function table further allowed students to explore the relationship between graphical, algebraic and numeric representations. Though students could clearly communicate the relationship between these views by the end of the investigation, some students struggled to apply these concepts to real-world linear applications.

In order to facilitate the connection between rate of change, fixed values and real-world linear applications, my students and I used the Lists & Spreadsheets application to perform a series of linear regressions. At first, students were given samples of real world linear models ($r=1$) and asked to complete data tables by hand. For instance, one of the initial scenarios was “A small Domino’s cheese pizza costs \$5 and each additional topping costs \$1.50.” Students then entered the data into the Lists & Spreadsheets application, performed a linear regression, and discussed the relationship between the m and b values and the original problem. After students became more comfortable with linear regressions, the examples grew more sophisticated. The final example was “In NYC, taxi fares are mandated by the NYC Taxi and Limousine Commission. It costs \$2.50 to enter a cab, and an additional \$0.10 per each block driven.” Students created a Lists & Spreadsheets table by hand, predicted the equation and verified their equations through a linear regression. In order to emphasize the power of writing an equation, I asked students to add a new Lists & Spreadsheet, add the input data by typing the appropriate name (“blocks”) and type “ $=.10(\text{blocks})+2.50$ ” into the output column. Students loved seeing the output column fill in automatically and, as a result, students discovered the efficiency of writing an equation.

Once students understood the relationship between the fixed and variable terms in the equation and the fixed and variable costs in the original problem, they created a scatter plot, graphed the linear regression as $f(x)$, measured the slope of the line, plotted the y-intercept and manipulated the line according to a series of questions. For instance, "How would the equation and graph change if the city changed the base rate from \$2.50 to \$3.00?" Since the data points from the scatter plot were fixed, students were able to compare the dynamic $f(x)$ to the original given information. Finally, students were asked to use the Calculator application to calculate the cost of taking a cab from the northern tip of Manhattan to the southern tip (about 250 blocks). Students then verified their solution by entering it into the spreadsheet and tracing it on the graph.

I was very pleased with students' level of understanding at the end of this activity, and I think this is largely due to using linear regression as a tool for inquiry. It allowed students to observe patterns between fixed and variable costs and relate them to the fixed and variable terms in an equation. Dynamically linked graphs and equations provided further grounds for discussion, and they allowed students to efficiently generate new examples based on modifications to the original problem situation. As a result of this activity, most students were extremely comfortable with modeling linear relationships numerically, algebraically and graphically.

He concludes:

Multiple representations also make fundamental concepts accessible to all learners. Instead of looking at four different problems over the course of a lesson (which was often the case before the TI-Nspire™ technology), my students and I will look at one problem in four different ways. Different representations address different learning styles, and both low and high-performing students benefit from this process. One or two representations may appeal to lower-functioning students, while high-performing students often deepen their understanding through one or two additional views (three and four representations on screen) of a given concept. As a result, I have observed a greater range of problem-solving techniques and more thorough verifications of work.

The immediate availability of a powerful learning handheld device proved important in improving the dialog among students and greatly facilitated spontaneous mathematical inquiry. Mr. Butterbaugh observed:

Dynamic investigations which utilize Graphs & Geometry, Lists & Spreadsheets and CAS (Computer Algebra System) capabilities allow students to actively identify patterns, make conjectures and formulate their own mathematical concepts. Often these conjectures are informally made between students and **the quality of student questions, to me as the instructor and to each other in cooperative settings, has dramatically increased.** Students have become more self-sufficient as a result of this communication, especially since these dynamic investigations often make concepts accessible to all levels of learners.

Another advantage of TI-Nspire CAS technology is that **we, as a class, are always a few steps away from clarification of common mathematical misconceptions**, especially those relating to numeric and algebraic skills. For instance, during certain lessons it becomes clear that some students have difficulty understanding that $(1/2)x$ is the same thing as $x/2$. In less than a minute, students can evaluate the validity of $(1/3)x=x/3$, $(1/4)x=x/4$, and $(1/5)x=x/5$ (all are "true" according to the Calculator app), make a conjecture and informally evaluate their conjectures by evaluating further arguments. Not only has TI-Nspire CAS technology clarified a common algebraic misconception, but it has done so efficiently and in an inquiry based manner. Similarly, dynamic graphs and/or geometry can also be used to quickly clarify common questions or misconceptions

The ability of TI-Nspire CAS technology to create and save documents proved important to Mr. Butterbaugh's teaching in a unit studying the general characteristics of linear, quadratic and absolute value functions. The results were dramatic.

Early in the school year, my 9th grade class was studying the general characteristics of linear, quadratic and absolute value functions. Traditionally, students struggle to connect the textbook definition of a function to its multiple views (representations on screen). So for one of our first TI-Nspire activities, I designed an exploration which

utilized dynamically linked representations to help students make connections between the multiple views of a function. As I experimented with the various ways to explore the concept, I decided that instead of using valuable class time (and patience) to construct a complex document with my students, I would provide them with a preconstructed (electronic) document to investigate and build upon.

The TI-Nspire™ document consisted of a variety of dynamically linked problems and was loaded onto the [TI-Nspire CAS] handhelds prior to class using TI-Nspire Computer Link Software. In the past, students have quickly applied the vertical-line test to a graph to determine whether or not it was a function, but they have consistently struggled to determine whether a relation in a table is a function. Therefore, the first few (document) pages each consisted of a split-screen with a Lists & Spreadsheet app (on the left of the TI-Nspire CAS screen) that was dynamically linked to the Graphs & Geometry app (on the right). Students entered a relation into the spreadsheet and the discrete data points appeared on the scatter plot. The scatter plot also contained a draggable vertical line which students could use to determine whether the relation was a function. For those relations that were not functions, students were asked to change the values in the table to make them functions. Students were then able to verify their changes by dragging the vertical line, which helped them further make the connection between the table and the graph.

Students then explored the graphs and equations of various relations. Using the draggable vertical line, students identified the characteristics of the equations in which graphs did and did not pass the vertical line test. Towards the end of the exploration, students encountered split-screen pages with the Notes application (on the right of the screen) and a blank application on the left. Students were asked to define a function and use this definition to generate their own examples. In the past, students have struggled to even restate the definition of a function in their own words. With these pre-constructed dynamically linked representations, however, students were able to define a function in their own words, generate examples of functions and their various views, and better articulate their reasoning throughout the entire process.

Though I was pleased throughout the course of this investigation, I was even more pleased with their performance on the exam which they eventually took over this material. The exam consisted of a variety of multiple choice and free response questions from the Math A (9th and 10th grade) and Math B (11th and 12th grade) curricula. Many of the questions included a comparatively large amount of writing and students did not have access to their TI-Nspire CAS handhelds during the exam. Despite the difficulty of the exam, the average score was 80% (it is normally in the low 70 percentile), and half of the class received an A or B grade.

I attribute much of this success to the fact that the students investigated a document which was largely preconstructed. Unless the purpose of a TI-Nspire activity is for students to perform some type of geometric construction, it is often more effective to provide a pre-constructed investigation or template. This is especially the case when an investigation requires a more advanced geometric construction with multiple parent-child relationships, as these types of steps can be difficult to justify to a room full of algebra students. The use of preconstructed documents allows us to maximize class time and focus on concepts instead of advanced constructions.

In general, Mr. Butterbaugh and most of his students reached proficiency with TI-Nspire technology quickly. He reports that he fully understood all the TI-Nspire CAS handheld's functions after approximately three weeks of use, and his students were able to use all representation modes and create documents after approximately five weeks of use. He also found that the laptop version of TI-Nspire CAS computer software had advantages for activities involving considerable typing and provided the flexibility of a larger viewing area on screen. The TI-Nspire CAS handheld was more efficient for mathematical operations and provided advantages for spontaneous use in class. Overall, Mr. Butterbaugh believes the TI-Nspire CAS handheld is the more effective classroom tool because it is more responsive to the daily demands of the classroom.

He recommends that teachers have a well-defined plan for managing documents, with specific and frequent "TI-Nspire CAS technology etiquette" instructions for students. Each student has to be assigned a specific unit and should have his or her own My Documents folder. Students need to be reminded to start and end the class by exiting the My Documents folder (before turning off the technology) to make sure that the next student does not inadvertently add pages to a

document in the previous student's folder. Mr. Butterbaugh has experienced few problems of students inadvertently erasing or deleting a fellow student's pages or documents.

He also points out that TI-Nspire™ CAS technology gives students the freedom to explore and that it is important to design lessons which encourage this freedom.

Just as good lessons and great lessons can be planned without TI-Nspire CAS technology, good lessons and great lessons can be planned with TI-Nspire CAS technology. Though part of the ability to distinguish between the two comes from experience, future TI-Nspire CAS users may benefit from discussions about the characteristics of good and great lessons. For instance, it took me a couple of weeks to learn that while clear and concise directions are important, too many directions - either verbally or in print - can turn an investigation into a scripted activity, which may ultimately eclipse a sense of freedom and purpose. As a result, I soon turned to preconstructed documents which maintained a healthy sense of simplicity and gave students the freedom to investigate. Effective questioning on the part of the instructor also plays an important role in student-centered investigations. I am still learning how to preserve the spirit of freedom while guiding student discovery with meaningful questions. Future TI-Nspire CAS users may benefit from discussing the types of questions that can be asked at critical points in a given lesson.

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