# Why should mathematics educators consider using technology with Computer Algebra System (CAS) features?

Research Note 10

Prepared for Texas Instruments by the Center for Technology in Learning, SRI International, December 7, 2007

*Some mathematics tools can input symbolic expressions, but output only numbers or graphs. CAS technology, however, can also output symbolic mathematical expressions. Researchers recommend that teachers use CAS features to focus on concepts, personalize the curricular sequence to fit student needs, and emphasize meaningful mathematical tasks. Although we await evidence based on the strongest research designs, studies throughout the world consistently report benefits when teachers integrate CAS with a focus on learning math concepts.*

### CAS allows teachers to emphasize "How, Why, and What if?"

In secondary school mathematics, the emphasis should shift from calculating particular numbers to using mathematics symbolically to analyze, prove or problem solve. Using mathematics symbolically is not just about "how" to do a particular algebraic step. It should also be about "why" and "what if?"

With CAS, students can perform mathematical operations on symbolic expressions to address these questions. For example, a teacher can ask a CAS calculator to "subtract x" from each side of the equation "$x + 10 = 3x - 20$" resulting in "$10 = 2x - 20$." The teacher can then focus on *why* this is an important step in solving the equality, rather than *how* to perform the step. (Of course, both "how" and "why" are important, and the teacher can decide not to use CAS features when focusing on the "how.")

Similarly, a teacher can form new functions related to the parabola $f(x) = x^2$ by "adding 5" or "adding x". All three functions can then be graphed, leading to a discussion of how adding a number or adding an "x" changes the graph of a parabola. The teacher is focusing on "what if?" and not on the mechanics of graphing.

Below we describe three keys to appropriate use of CAS, and then summarize existing evaluative research.

### 1. Use Efficiency Gains to Focus on Mathematics Concepts

CAS performs manipulations accurately and quickly. Students can obtain both exact and approximate results without worrying about tedious steps and errors. Researchers argue that teachers can use the efficiency gained from using CAS to focus on conceptual development, problem solving and investigations with realistic problems (Heid, 1988; Hillel, 1993). Furthermore, they suggest that weak students can greatly benefit from the accuracy and immediacy of CAS. Instead of getting stuck in routine procedures, students can experience a more complex task such as making connections between an algebraic expression and a graph (Kuzler, 2000).

Researchers (e.g., Heid, 1988; Porzio, 1999; Kaput, 1996) have also argued that CAS supports students' exploration of problems and concepts through manipulating algebraic expressions in multiple forms of representations (e.g., numerical, graphical, and symbolic). A teacher can help students make sense of mathematics and learn strategies to develop "algebraic insight" by focusing more on interpreting and reflecting than on simply carrying out steps of procedures (Arnold, 2004; Pierce & Stacey, 2007).

### 2. Personalize the Curricular Sequence for Student Needs

With CAS, teachers have more flexibility in the curriculum sequence. They can teach concepts and applications before algebraic operations, an approach not traditionally possible (Heid et al., 1988). Referring to CAS, Day (1993) enthusiastically stated "the power and flexibility of technology can help change the focus of school algebra from students becoming mediocre manipulators to their becoming accomplished analysts" (p.30). Teachers can take advantage of these capabilities to personalize the curricular sequence to fit student needs.

### 3. Emphasize Pedagogically Meaningful Tasks

As with any learning tools, research shows that the mere presence of CAS does not guarantee better student learning. In reviewing empirical studies published up to 1995, Mayes (1997, cited in Hoyles & Noss, 2006) found less positive outcomes from the cases where CAS was used primarily for increasing efficiency and speed in implementing traditional approaches to problem solving. While researchers continue to debate details about where and how much CAS should be used (Bohm et al, 2004), they unanimously agree on the importance of teachers as agent of integration and change. For example, to avoid superficial, unproductive use of CAS, teachers must give students tasks that have clear and valid pedagogical and functional benefits of CAS (Pierce & Stacey, 2004; Ruthven, 2002).

## Research Shows Gains in Student Achievement and Motivation

Since the late 1980s, researchers worldwide have consistently reported positive learning gains from classrooms that integrated CAS appropriately. While not all studies involve a strong research design or large sample, the evidence accumulated over two decades from a variety of countries and settings is noteworthy.

Early classroom experiments of CAS demonstrated that students who used CAS and experienced concept-oriented lessons achieved significantly greater understanding in conceptual knowledge than those students in traditional skills-oriented conditions, without sacrificing the learning of computational skills (Heid, 1992; Heid, 1988; Palmiter, 1991).

More recent studies have generally confirmed this trend. A large study involving hundreds of college students showed that when teachers used CAS technology with an appropriate instructional emphasis (i.e., on making sense of mathematics in group discussions), students learned to reason about symbolic expressions (Keller & Russell, 1997). The students who were taught with CAS were more successful than students without CAS at three levels: basic computation, more advanced computation and complex symbolic problems. Similar findings were reported for upper secondary school students (ages 16-19) in Finland who were taught the concept of derivative with and without CAS (Repo, 1994).

Research has also examined student motivation. When used effectively, CAS can make math more interesting and meaningful to students. A study with Greek business students showed that students using CAS were more interested, participated actively and spent more time preparing for class than their non-CAS-using counterparts (Vlachos & Kehagias, 2000). Similarly, a recent 3-year study of CAS with grade 11 and 12 students in Germany found a moderate enthusiasm for mathematics and overall positive attitude to CAS (Schmidt and Moldenhauer, 2002).

**References:**


Arnold, S. (2004). Classroom computer algebra: Some issues and approaches. *Australian Mathematics Teacher, 60*(2), 17-21.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning 7*(3), 275-291.

Ball, L. and Stacey, K. (2005). Teaching strategies for developing judicious technology use. In W. J. Masalski & P. C. Elliott (eds.), Technology-Supported Mathematics Learning Environments (2005 National Council of Teachers of Mathematics Yearbook, pp. 3-15). Reston, VA: NCTM.

Day, R. (1993). Algebra and technology. *Journal of Computers in Mathematics and Science Teaching, 12*(1), 29-36.

Drijvers, P., and O.A. van Herwaarden (2001). Instrumentation of ICT-tools: the case of algebra in a computer algebra environment, *The International Journal of Computer Algebra in Mathematics Education,* 7(4), 255-75.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education, 19*(1), 3-25.

Hillel, J. (1993). Computer algebra systems as cognitive technologies: Implications for the practice of mathematics education. In C. Keitel & K. Ruthven (Eds.), *Learning from computers: Mathematics education and technology* (pp. 18-47). Berlin: Springer-Verlag.

Hoyles and Noss (2006) What can digital technologies take from and bring to research in mathematics education? In A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick and F.K.S. Leung (eds). *Second International Handbook of Research in Mathematics Education* (pp. 323- 349). Dordrecht: Kluwer.

Kaput, J. (1996). Algebra and technology: New semiotic continuities and referential connectivity. In F. Hitt, T. Rojano, & M. Santos (Eds.), *Proceedings of the PME-NA XXI Annual Meeting*, Cuernavaca, Mexico.

Keller, B. A., and Russell, C. A. (1997). Effects of the TI-92 on calculus students solving symbolic problems. *The International Journal of Computer Algebra in Mathematics Education, 4*(1), 77-98.

Kuzler, B. (2000). The algebraic calculator as a pedagogical tool for teaching mathematics. *International Journal of Computer Algebra in Mathematics Education.7*(1), 5-23.

Lagrange, J. B. (1999). Complex calculators in the classroom: Theoretical and practical reflections on teaching pre-calculus. *International Journal of Computers for Mathematical Learning, 4,* 51–81.

Mayes, R. (1997). Current state of research into CAS in mathematics education. In Berry, J., Kronfellner, M., Kutzler, Monaghan, J. *The State of Computer Algebra in Mathematics Education* (pp 171-189). Chartwell-Bratt, Bromley.

Palmiter, J. R. (1991). Effects of computer algebra systems on concept and skill acqisition in calculus. *Journal for Research in Mathematics Education, 22*(2), 151-156.

Pierce, R. and Stacey, K. ( 2007) Developing algebraic insight. *Mathematics Teaching 203*, 12-16.

Pierce, R. and Stacey, K. (2004). A framework for monitoring progress and planning teaching towards effective use of computer algebra systems. *International Journal of Computers for Mathematical Learning, 9*, 59–93.

Porzio, D. (1999). Effects of differing emphases in the use of multiple representations and technology on students' understanding of calculus concepts. *Focus on Learning Problems in Mathematics, 21*(3), 1-29.

Repo, S. (1994). Understanding and reflective abstraction: Learning the concept of derivative in a computer environment. I*nternational DERIVE Journal, 1*(1), 97-113.