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Examining Students' Perceptions of Graphing Calculator Use During Instruction and Problem-Solutions Following Technology-Enhanced Instruction

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Abstract

This exploratory study investigated the effects of instruction enhanced by the TI-Nspire on student achievement, students' perceptions of technology use during instruction, problem-solving success, problem-solving solution strategies, and the relationship between the number of strategies students used to solve a problem and success. Four Algebra II classrooms (N=53) were randomly assigned to use the TI-Nspire or TI-83+. Data were collected using a survey and teacher-created unit test. Students' solution strategies were associated with treatment ($p = .033$). Students in the TI-Nspire group tended to use more graphical representations, and those in the TI-83+ group tended to use symbolic solutions. Furthermore the number of strategies used to solve the problem was related to success ($p = .001$). Students in the two groups did not significantly differ on a teacher-developed unit test, problem-solving success, and number of strategies used to solve a word problem. This study provides preliminary evidence that technology-enhanced instruction can influence students' use of multiple representations while solving mathematical word problems.

Examining Students' Perceptions of Graphing Calculator Use During Instruction and Problem-Solutions Following Technology-Enhanced Instruction

The graphing calculator is a cognitive tool that supports problem-solver's critical thinking and engagement in authentic mathematics activities that foster learning mathematics at a deep conceptual level (NCTM, 2000). Graphing calculators provide efficient means of computation, graphical displays of mathematical relations, access to tables of values, facility to write executable code, and the most current technology provides students quick access to multiple representations of mathematical relations. Using this tool, students are able to efficiently graph mathematical relations and become engaged in constructing meaning, for example, by creating and examining a table of values and inferring relationships without performing more mundane processes prior to engaging in the thinking necessary to construct these relationships. Handheld graphing technology has been a relatively recent addition to mathematics classrooms (Hirschorn & Thompson, 1996; Heid, 1997; Heid & Zbiek, 1995; Palmiter, 1991), and research has begun to produce evidence supporting their use to enhance students' understanding of mathematics (Artigue, 2002; Heid & Edwards, 2001; Herman, 2007; Pierce & Stacey, 2001).

Too often, however, students use paper-and-pencil algebraic approaches exclusively to solve mathematics problems without considering other representations or solution strategies that are as accurate and sometimes more efficient (Herman, 2007; Knuth, 2000). The purpose of this exploratory study was to investigate students' perceptions of technology use within their Algebra II class and the effects of technology-enhanced instruction on success and students' use of a variety of solution strategies to solve a complex word problem.

A Case for Multiple Representations

The value of multiple representations for problem solving within mathematics classes has a foundation in psychology and education (Brenner et al., 1997; Greeno & Hall, 1997). Representations are tools students use while learning mathematics and during mathematically-relevant classroom communication. Without knowledge of multiple representations, students may have difficulty connecting mathematically-related ideas. Learning opportunities in the mathematics classroom that incorporate representations support learners' development of cognitive flexibility necessary to shift between mathematical representations. Greeno and Hall (1997) discuss a classroom example in which middle school students were required to build mathematical models of prey-predator populations using computer software. Students receiving instruction using multiple representations discovered that these different representations provided access to different inferences and calculations. This group of children easily transitioned between an equation and data found in an accompanying graph and table. They started with an initial equation and then examined the graph and table, which led to further modification of their equation. This adjustment cycle continued until the biological model was adequately explained. Students used mathematical representations to efficiently meet a goal and recognized the utility of multiple representations when solving subsequent algebraic function problems.

Within an experimental context, Brenner et al. (1997) investigated 128 pre-algebra students' use of multiple representations to solve function problems following instruction incorporating multiple representations. Students were asked to solve questions such as, "Mary Wong just got a job working as a clerk in a candy store. She already has $42. She will earn $7 per hour. How many hours will she have to work to have a total of $126?" (p. 671). Problems such as this can be solved using several solution strategies such as graphical, symbolic, or tabular approaches. Students engaged in instruction emphasizing multiple representations were more likely to succeed in finding the problem's solution and performed better than their peers on mathematical tests of functions presented in a word problem format. Students who did not practice using representations did not consider alternative representations other than their initial representation. Learners need time and exposure to various mathematical representations in order to select and correctly employ a representation while solving a problem. This exposure needs to come during the course of instruction that models how to use, adapt, and construct mathematical representations (Greeno & Hall, 1997).

Despite curricula and instructional programs that highlight multiple representations in middle school, many students leave secondary mathematics classrooms with the notion that mathematical representations are not linked and they are uncertain when to use a representation to meet a desired learning goal or problem solution. Knuth (2000) investigated 178 secondary students' ability to use, select, and move between algebraic and graphical representations of functions. Classroom teachers presented students with mathematics problems that required learners to recognize that a point is located on a graph of line L if and only if its coordinates satisfy its equation. Questions probed students' mathematical knowledge and ability to provide solution strategies using multiple representations. Students' choice of representation affected the efficiency of solution but not the solution. The majority of participants chose algebraic approaches that were less efficient than a graphical approach and many students failed to recognize that a graphical solution strategy was viable. Students had difficulty selecting solution strategies using non-algebraic approaches and even more trouble shifting between representations. In the present study, we examine the impact of the graphing calculator on students' use of multiple solution strategies, which is the focus of the literature reviewed in the next section.

Graphing Calculator Technology Enables the Use of Multiple Representations

Research compilations characterizing effects of graphing calculator technology on student-related outcomes illustrate the positive impact of these tools (Heller, Curtis, Jaffe, & Verboncoeur, 2005; Interactive Educational Systems Design (IESD), 2003). The IESD report examined five studies meeting criteria for scientifically-based research on the impact of handheld graphing calculators within Algebra courses. The authors caution at least one study was confounded with the use of reform-based curricula; however, these studies depict overwhelmingly positive outcomes on students' understanding of Algebra concepts when incorporating handheld technology. Similarly, Heller et al. (2005) investigated the effects of students' access to graphing technology on achievement scores. An increase in students' mathematics achievement scores was positively related to calculator technology access. Mathematics teachers who provide instruction that uses the graphing calculator will enhance students' development of critical mathematics concepts.

Algebra students encounter many opportunities to employ graphical, tabular, or other approaches when learning about functions. "Representation is pervasive in algebra. Graphs convey particular kinds of information visually, whereas symbolic expressions may be easier to manipulate, analyze, and transform" (NCTM, 2000, p. 360). Conceptually, quadratic functions provide learners an opportunity to engage in multiple representational solution-strategy approaches. For example, given the equation $y = -16x^2 + 688x + 0$, a student might choose to solve the quadratic function using a symbolic heuristic such as the quadratic formula, completing the square, factoring, or guess and check. A graphical representation supports locating values of the independent variable when the dependent variable equals zero. Use of a tabular strategy suggests inspecting specific dependent and independent values or considering trends within the list of values.

Students’ application of symbolic strategies reflects their perceptions that doing mathematics entails symbol manipulation (Herman, 2007; Knuth, 2003). Herman investigated the impact of the TI-83 graphing calculator on students’ choice of problem-solving strategies and learners’ facility with multiple representations. Thirty-eight college-age algebra students took a six-item pretest and posttest to demonstrate their proficiency with multiple representations before and after ten weeks of mathematics instruction that used the graphing calculator. While students struggled to construct an understanding of polynomial functions, those who used more than one representational approach tended to answer more items correctly. Participants reported that knowing more than one strategy was helpful, but, despite exposure to technology-enhanced instruction were reticent to use the calculator and preferred purely symbolic approaches. The calculator merely provided these students a means to check computational accuracy.

The graphing calculator offers students an opportunity to shift their focus from less important matters and toward a complex task that requires more cognitive energy. As discussed earlier, fostering opportunities to develop connections between representations is important for supporting students’ development of a deep understanding of mathematics concepts (Boaler, 2003). Technology use and student-centered activities support students’ connections among mathematically relevant ideas and potentially shape students’ views of mathematics as a discipline (Ruthven, 2002). Students need ample time to practice utilizing graphing calculator technology while solving mathematical problems (Burrill, Allison, Breaux, Leatham, & Sanchez, 2002; Herman, 2007), and with increased time using the graphing calculator, students typically come to view technologies as more than computational devices (Herman, 2007; Ruthven, 1995).

The present study examines the effect of a unit of instruction related to rational functions on students' achievement on a teacher-developed test, students' perceptions of technology use during instruction, problem-solving success, problem-solving solution strategies, and the relationship between number of strategies and success for two groups of students. One of the groups was allowed to use the TI-Nspire graphing calculator while the second group experienced instruction using the TI-83+ graphing calculator.

Method

The study took place in an Algebra II classroom in a city located in Florida. Three school faculty including the Algebra I teacher, Algebra II teacher, and the mathematics department chair co-planned the lessons with a graduate student in mathematics education and a mathematics education faculty member. During the study, the graduate student provided feedback to teachers on proposed lessons and technology-related instructions. We began work on lesson plans in late January 2008 and teachers implemented lessons during a three-week period in the second semester. For their support, their school received one class set of TI-Nspire calculators to retain for future use as well as reimbursement for participation at the Texas Instruments international conference.

Participants

The Algebra II teacher, whom we will refer to as Emma, has over twenty years of experience teaching mathematics and holds a Master's degree in Mathematics Education and an Educational Specialist's degree in Curriculum and Instruction with a minor in Special Education. She taught four Algebra II classes with approximately thirty students per class. Two of these sections were considered honors sections. All Algebra II students were invited to participate. Fifty-three out of 106 students returned completed assent and consent forms and were eligible for the study. Thirty students participated in the TI-Nspire (Treatment) group, and 23 participated in the TI-83+ (Control) classes. The school's demographics are representative of the state of Florida with 24% African American, 3% Asian, 51% Caucasian, 16% Hispanic, and 5% multiracial, and an equal proportion of male and female students attending the school.

Instrumentation

Student achievement was measured by a teacher-developed unit test based on content and problem types taught during the unit. Students were required to factor quadratic expressions, find the roots of quadratics equations, simplify and perform operations with complex numbers, and graph quadratic equations.

Students' perceptions of technology use during instruction, problem-solving solution strategies, and problem-solving success were captured using a survey about the use of the graphing calculator within their classroom and algebraic problem solving. Students were asked to report their use of the graphing calculator and their perceptions of whether the technology supported their understanding during the unit. In addition, students solved a quadratic-relation problem demonstrating as many strategies as possible. The final question challenged students to solve a linear function problem, but due to time limitations few students provided data in response to this question, and it was eliminated from further analysis.

The quadratic-related problem was similar to the problem used in Herman's (2007) study and could be solved using algebraic, graphical, or tabular strategies or by calling upon and executing calculator programs. One might choose to factor the expression following setting the quadratic expression equal to zero or use the quadratic formula to find the roots of the equation.

A graphical or tabular approach would encourage a search for x-values when y equals zero. In addition, the CSOLVE program on the TI-Nspire graphing calculator may be used to perform these procedures.

Procedure

The study was conducted during one unit of instruction approximately three weeks long. Students typically met with their teacher during a 90-minute period two days per week and a 45-minute class one day per week. We randomly assigned one Algebra II and one Algebra II honors section to have access to TI-Nspire technology and the remaining classes were provided access to the TI-83+ graphing calculator only. Students had access to the respective calculators for use during in-class activities, projects, assessments, and in-class homework completion. Further, students in the TI-83+ group who owned their own calculators had access outside of class. The survey was given on the final day during instruction whereas the teacher administered the achievement test the following class.

All class sessions were observed and videotaped. A typical class meeting in either condition began similarly: the teacher distributed calculators, the students completed a warm-up problem, and the teacher reviewed the previous night's homework assignment. In the treatment condition, the teacher provided guided instruction through activities supported by the use of the TI-Nspire, which tended to take sixty minutes of the class period. The teacher typically allowed students to complete each activity on their own or in pairs after which students were allowed to freely explore the calculator. Students who discovered novel ways to use the calculator were encouraged to share their findings with the class. Near the end of class, the teacher collected calculators from students and assigned homework due the following class. In the control condition, students experienced instruction that was traditional, teacher-centered, and less calculator-based.

The survey was administered during a 25-minute session during a one-week period, and the teacher-developed unit test was completed during a subsequent class period. Participants were provided access to paper, pencil, graph paper, and calculators (i.e., TI-Nspire or TI-83+ depending upon their treatment group) while completing the survey and unit test.

Data Analysis

Student test scores were submitted to an independent samples t-test analysis to examine differences between treatment and control groups. Student perceptions of their use of the graphing calculator and its effect on their understanding were coded by both authors. Students' uses of the technology were categorized supporting understanding, facilitating procedures, or not relevant. Students' perceptions of whether use of the technology supported their understanding were coded as positive, neutral, or negative. Chi-square analyses were conducted to examine relationships between treatment condition and student responses.

Students' problem solutions were coded as correct or incorrect, and treatment and control students' problem-solving success was examined using an independent samples t-test. Students' problem-solving approaches were categorized by two coders into several categories: graphing (by-hand and calculator), calculator-based programs, tabular (by-hand and calculator), symbol manipulation (by-hand only), or no response. Calculator-based programs were not available to students in the control group, therefore these responses were omitted from further analysis. The number of strategies each student used was tallied. The relationship between treatment group and types of strategies as well as number of strategies was examined separately using a chi-square analysis. Finally, the relationship between the number of strategies used and problem-solving success was examined using an ANOVA with a Scheffé's post hoc analysis.

Results

The purpose of this exploratory study was to investigate the effect of using the TI-Nspire on students' achievement, students' perceptions of technology use during instruction, problem-solving success, problem-solving solution strategies, and the relationship between number of strategies and success. To do this, classes of students were randomly assigned to the treatment (i.e., use of the TI-Nspire) or control (i.e., use of TI-83+) conditions.

To investigate differences in achievement between students using the TI-Nspire and TI-83+ we compared the treatment and the control groups' outcomes on the teacher-developed unit test. TI-Nspire and TI-83+ students performed similarly on the unit test, $t(56) = 0.50$, $p = .62$ ($M_{RX} = 77.59$, $SD_{RX} = 10.62$; $M_C = 79.12$, $SD_C = 12.42$).

Students in the treatment and control groups did not report using the technology differently in their instruction ($\chi^2(2, N = 52) = 1.51$, $p = .47$) nor did they report differential support for understanding following use of the two technologies ($\chi^2(2, N = 52) = 1.51$, $p = .47$). Statements from students using the TI-Nspire, however, provide insight into their experiences. Students' statements indicate a slightly greater number of students in the TI-Nspire group than expected reported using the technology for understanding, and their responses were more elaborate than their control group counterparts. When asked whether the TI-Nspire supported their learning, one student reported, "Yes, it extended my horizons in ways nothing else could. With it, I discovered for myself the effect of having a squared variable in a cubic polynomial in less than five minutes. Graphing by hand, this endeavor would have taken hours to reach the same conclusion." Other students in this category indicated that the calculator supported them to carry out procedures more efficiently. Students in both groups indicated the usefulness of the technology relative to learning procedures such as graphing. “The massive amount of functions in this calculator have made doing homework much easier in this class. Specifically, the ‘CSOLVE’ and ‘ZEROS ()’ functions come in very handy for finding the zeros on the x and y axis [*sic*]. The catalog button is also very handy for finding any function. Getting rid of the alpha key and extending the letter keys made equation typing easier.” Several students indicated that the technology supported their learning because the technology helped them to “break down” the procedures: “Yes it did [support learning] because it helped me brake [*sic*] down the problem play by play because we could go back.” Further, additional students indicated that the TI-Nspire helped them to visualize or “see” the form of a graph. “I used the graphing calculator to find out what graphs look like, problems like mx + b and finding out what the true vertex is and axis of symmetry.” Yet another student reported that “[he or she] used the graphing calculator over the past few weeks to plug in equations and such to see the graph. I also used it to see the zeros of an equation that I couldn’t find.”

Students in the TI-Nspire group who responded negatively in terms of the calculator’s support for understanding reported limited facility using this complex tool as their rationale. For example, “Yes [the Nspire did support my understanding], because it showed me the steps to the questions. But no, because the Nspire is so advanced from my 83 that it took a little while to figure out how to do it on the calculator.” Another student reported, “I don’t feel like it helped me understand Algebra any better than I did before. In order to use this calculator proficiently you need to understand Algebra beforehand, so you know what you’re typing into the calculator. I already understand the things we did on the calculator, it was just a matter of figuring out how to actually do it on the calculator.”

While not statistically significant, there was a trend toward a difference between success rates on the quadratic-relation problem, $t(51) = 1.93$, $p = .059$ ($M_{RX} = .70$, $SD_{RX} = .47$; $M_C = .43$, $SD_C = .50$). Students in the TI-Nspire group were more successful solving this problem than those in the control group. In addition, there was a significant relationship between the types of strategies used and treatment condition, $\chi^2(3, N = 73) = 8.74$, $p = .033$. A visual inspection of Table 1 indicates that students in the TI-Nspire group were more likely to use graphical representations than students in the TI-83+ group. Conversely, students in the TI-83+ group were more likely to choose symbolic strategies than TI-Nspire students. There was no relationship, however, between the number of strategies used to solve the problem and treatment group, $\chi^2(2, N = 53) = .83$, $p = .66$ (Table 2). Importantly, there was a relationship between the number of strategies used to solve the word problem and success regardless of treatment group assignment, $F(3, 49) = 6.52$, $p = .001$. Scheffé's post hoc analysis indicated that students who used three strategies were more successful than those who used only one strategy to solve the word problem, and students who used two or more strategies were significantly more successful in solving the problem.

Insert Tables 1 & 2 about here

Conclusion

This exploratory research provides some evidence for the benefits of using the TI-Nspire within an Algebra II classroom. While there were no benefits to students in overall achievement on the teacher-developed unit test nor differences between students' perceptions of the use of the technologies or its benefit for their understanding, affordances were evidenced in the students' strategic behavior and outcomes on the quadratic-relations word problem. Specifically, students in the TI-Nspire group used more varied solution strategies and were more successful solving this problem. That is, these students were more likely to use several different strategies while the TI-83+ students tended to favor purely symbolic strategies. Herman's (2007) results were similar despite instruction using a graphing calculator. While students in the TI-83+ group responded with behaviors similar to those found by Herman, students in the TI-Nspire group used symbolic strategies less frequently. Thus, the results of the present study begin to provide evidence that the TI-Nspire may support changes in students' solution strategies. Previous studies linked use of multiple representations to improving students' beliefs about mathematics and students' problem-solving ability (Brenner et al., 1997).

The TI-Nspire is unique from other graphing calculator technology as it allows students to see more than one representation in split-screen mode. For instance, students are able to look at the graph of a function on one half of the viewing window while simultaneously examining dependent and independent values. The TI-Nspire may foster cognitive linkages between mathematical representations because of its display features. As one student eloquently noted:

> The calculator is simply stunning. Its built-in databank of possible operations feels limitless and allows the user to perform operations not possible on earlier models. The multiple windows allow the user to flow seamlessly between calculations, graphs, notes, and other function[s] I haven't even explored yet. With this tool, the only stumbling block left to users is their own comprehension of subject matter.

Using older graphing technology, students can view the graph in one window and then press a few buttons to investigate dependent and independent values found in a table. While these representations are available on the earlier technology, the learner must exert more cognitive effort to construct conceptual links. The graphical interface of the TI-Nspire facilitates this process by presenting these different representations adjacent to one another.

While we hold the results of the present exploratory study tentatively, they provide insight and impetus for future research. This limited sample of students was able to use more than one representation to meet a goal and was less likely to choose typical symbolic approaches to solve a quadratic-relation problem. Mathematics educators continue to investigate the mechanism for supporting students to use multiple approaches beyond the traditional symbolic approaches since other representations (e.g., tabular and graphical) are often more efficient and less tedious. Future research with larger samples of students and more expansive instrumentation are warranted. The results of the present study, however, suggest the possibility that the technologies teachers provide to students potentially influence their use of multiple representations in the mathematics classroom.

Limitations

This study was conducted during one unit that lasted approximately three weeks. While the students had been using the TI-83+ for at least two years prior to this study and were comfortable using it, they found the TI-Nspire complex to use. This familiarity with the TI-83+ confounds the results since students in the TI-Nspire group were asked to familiarize themselves with the instrument while learning mathematics. Further, since this technology was novel to the students they did not have access to it outside of the classroom. Those in the TI-83+ group typically owned their own graphing calculator. Students may have needed more time inside and outside of class to familiarize themselves with the new tool. Burrill et al. (2002) reported similar findings; time spent using the calculator is a critical factor affecting student involvement in mathematics learning. In that study, students were better able to handle mathematics content learning after they mastered the use of the tool to meet their cognitive needs. Future studies should consider using a longer time frame to allow students adequate time to become familiar with the new technology.

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Table 1

Number (percent) of students employing various strategies to solve quadratic relation problem

Type of Solution Strategy	TI-Nspire	TI-83+
Graphical	24 (.47)	10 (.29)
Tabular	3 (.06)	3 (.09)
Symbolic	6 (.12)	14 (.41)
CSolve	12 (.24)	0 (.00)
No Response	6 (.12)	7 (.21)
Total Number	51	34

Note: Since CSolve was not available on the TI-83+, it was not included within the chi-square analysis.

Table 2

Number (percent) of students solving quadratic relation problem using different numbers of strategies

Number of Different Solution Strategies	TI-Nspire	TI-83+
None	6 (.20)	7 (.30)
One	11 (.37)	8 (.35)
Two or more	13 (.43)	8 (.35)
Total Number	30	23