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The Effects of the TI-Nspire CAS on Student Learning

A Phase I Project

by

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Introduction & Research Questions

In 2000 the National Council of Teachers of Mathematics (NCTM) released the *Principles and Standards for School Mathematics*. In this document one of the six major principles addressed by NCTM was technology. "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (NCTM, 2000, p.24). If you walk into a typical mathematics classroom you will see technology being used. Hand held technology is consistently growing and changing. So much so that now the ability to manipulate algebraic symbols is readily available. Systems that can manipulate algebraically are called Computer Algebra Systems (CAS). Although some may see symbolic manipulators as a means to check answers, there is a lot of research that supports that it does provide the opportunity for students to develop understanding of mathematical concepts through discovery (Heid, 2002; Leinbach, 2001).

I had the opportunity for my students to use this new TI-Nspire CAS technology and work on activities that utilize CAS technology. Many current studies on CAS include the use of graphical representations. Therefore results do not necessarily accurately show how CAS uniquely contributed to student improvement in achievement or understanding as graphing ability is available in non CAS handheld technology. Although CAS may be used as a graphical tool, it is its algebraic/symbolic manipulation ability that was the focus of this study. Thus, student use of TI-Nspire CAS was specifically geared towards the students' use of the symbolic manipulation features of the technology. The following research questions guided my pilot study:

- What are the effects of the use of the TI-Nspire Computer Algebra System (CAS) ?

- o Does the use of CAS technology have an effect on student learning of algebraic rules and symbolic manipulation?

- o Does the use of CAS technology help students to learn, retain, and understand procedural knowledge or gain procedural fluency?

- o Does the use of CAS technology help students gain understanding of mathematical concepts or impact their conceptual understanding?

- o Did the TI-Nspire CAS become an instrument that enabled students to learn mathematics?

- o How does the classroom structure (cooperative learning groups) help or hinder student progress during activities utilizing CAS technology?

- • What are student impressions of how the TI-Nspire CAS helped them learn mathematics?

  - o How useful students feel the TI-Nspire CAS was in helping them understand the material they were studying?

  - o What are some of the things students were able to do using the TI-Nspire CAS ?

Research Methodology

My goal was twofold. I was studying not only the effects in students learning of mathematics through the use of CAS, but also trying to understand my students' experiences with the TI-Nspire CAS technology.

One of the primary focuses for this pilot study was on student learning of mathematical concepts. One way in which I planned on looking at student learning of mathematical concepts, was by looking at my classroom from a design research perspective. Students worked in cooperative groups using the TI-Npire CAS to complete directed activities where they needed to

look at patterns and make conjectures and create rules based on those patterns. Central to the use of the technology was the type of activities being used as well as the expectation that students were writing their results and discussing their findings with their group mates. Therefore, my intent was not only to study the product, the TI-Nspire CAS, but the process, the group cooperation and participation in cooperative learning group activities. Therefore, one lens I intended on using in analyzing my data was viewing it as a design research study as my intent was to study the design of the learning environment simultaneously with studying student cognition/learning using the TI-Nspire CAS.

In order to look at the social process of learning in a CAS environment, students worked with the technology in cooperative groups where they discuss answers through exploratory activities. Thus, I was looking at the epistemological influences on student acquired knowledge through a social constructivist lens (Howe, 1998) as well as the effects of the design of the CAS exploratory activities used. I do not necessarily think that the only way to use this technology is in this type of setting, but in a Vygotskian way the CAS activities that were included in this research were designed were to use the CAS as a tool used to promote pattern exploration and thus provide the arena for student conversations which are mediated by the use of the CAS as a tool (Roschelle & Jackiw, 2000; Vygotsky, 1986).

In order to study the "product", the student cognition/learning using CAS, I approached the data from two different lenses. My initial intention was to perform a phenomenological analysis to try and gain understanding of how my students interacted with this new technology (Denzin & Lincoln, 2002; Patton, 2002) of CAS.

In order to analyze the data from this perspective, I needed to be very aware of my own biases. I have attended many conferences and have even presented at conferences using CAS technology. Also, I was able to perform my pilot study due to a grant that I received from Texas Instruments. Therefore it was critical that I made myself open to see what goes on in my classroom and make sure that I was ever aware of my own assumptions or viewpoints. I routinely went through this process of epoche in order to be able to investigate my students' use of this technology from a fresh and open viewpoint without prejudgment (Patton, 2002). This was important as I believe it prevented me from prematurely imposing meaning or jumping to conclusions hastily.

Next I used bracketing to elicit student reflections and interpretations of phrases used by themselves and classmates during class. These student reflections were gathered through transcriptions of video taped lessons, student journals, teacher journals, or focus group audio taped interviews. I looked for recurring features or reflections about the use of this technology. I then tried to put data into clusters to look for themes. My goal was to try and give an overall structural synthesis describing the experience of my students using the TI-Nspire CAS.

Finally for this phenomenography I performed a retrospective analysis where I looked again at all data sources: video recordings of classroom sessions, audio taped interviews with student focus groups, copies of students' written work, and the teacher journal in order to determine how the activities, the social structure of the class, and the CAS technology helped to support the mathematical development of the participating students.

In this study I was looking at the use of technology as a symbolic manipulator and the affects that having this tool and allowing students to use this tool would affect their learning.

Therefore, another lens with which I hoped to analyze my data was to look at the instrumental genesis of how and whether the TI-Nspire CAS as an artifact actually becomes an instrument that enabled students to learn mathematics. "Instrumental genesis includes both the user shaping the tool for her or his purposes (instrumentalization) and the user's understanding being shaped by the tool (instrumentation)" (Zbiek, Heid, Blume, & Dick, 2007)p. 56.

Participants

This study included students from a public high school in Northern Virginia. A total of two Algebra Geometry III classes participated in this study. This study included a total of 15 students, 6 female, 9 male, whose ages range from 16-18. Approximately 20% of students are black and 20% Hispanic. There are 3 students (20%) in the English as a Second Language learners (ESL) program. Only two Caucasian students asked not to participate in group interviews. The students in this study are in the third year of a three year program resulting in completion of both Algebra and Geometry credit.

Data Collection

The purpose of this project was to study the effects of the use of TI-Nspire Computer Algebra System (CAS) on student learning in mathematics classes. As part of regular classroom instruction all students have been using the TI-Nspire CAS. As part of regular instructional practices, students kept journals to document answers to activities using CAS as well as answers and comments from students on their understanding of concepts that were taught through activities designed using the TI-Nspire CAS. The teacher also kept a log which was used to document: the success of activities, suggestions for future implementation, changes noted in student learning. During the eight weeks of this study classes were being regularly video taped. Portions of video taped classes were used in analysis. Specifically, the teacher watched

videotapes to listen to students dialog while using the TI-Nspire CAS. The video was viewed to look for student interaction in small groups to see if the design of the social setting influenced student learning. Also, the researcher looked at individual comments from students which related to their understanding or use of the technology as well as those which demonstrated positive or negative effects of the TI-Nspire CAS on students learning.

Students who participated in this research project gave permission for existing data that had been collected (video taped lessons, journals, and student assessments) to be used by the teacher researcher for analysis purposes. There are no activities that were being conducted in class for research purposes only. All of the aforementioned activities were regular classroom instructional practices.

This research project was approved by the school district in which the research was being completed as well as by the human subjects review board (HSRB) for the graduate school where the researcher is pursuing an advanced degree in mathematics education leadership.

On their student assent forms students were given the option to agree or disagree to participate in a focus group interview. The focus group interviews were conducted in order to gather information about students' reactions to using this technology, as well as its perceived impact on their performance. Small group interviews were conducted during class time with groups of no more than three students. The group interviews were audio taped and the interviews were transcribed and coded for analysis to see if there were any changes in student learning or ability to learn through the use of the TI-Nspire CAS.

The researcher was the teacher for the student participants. Therefore to avoid any undue influence, permission to participate was not requested until after quarter 3 grades had been

posted.   Also, there are no activities that are being conducted for research purposes only. All classroom activities involving the use of the TI-Nspire CAS, including journal entries, and video taped lessons, are all part of the teacher researcher's regular classroom practices.

The informed consent form was distributed to students and discussed in class after 3rd quarter grades were posted. Permission required the student signature as well as the signature of their parent. The researcher-teacher disclosed the nature of this research with her students and distribute a cover letter (appendix B), parent consent forms (appendix C), and student assent forms (appendix D) in class for students to bring home to get signed. The researcher- teacher documented the students who submitted their forms. The researcher ensures that only data gather from students who had agreed to participate (and who were given permission to participate by their parent) were used in the analysis. Only students who willingly checked the box to participate in focus group interviews were approached to participate.

There was no compensation for participating in this study. Students did not get bonus points in class or any additional grades or course credit for participating in this study. Students who did not choose to participate were not penalized.

Analysis

Over the eight weeks of data collection, several videos were recorded of student work in problem solving settings. The researcher reviewed all tapes sequentially in order to not only look for ways groups work together using the technology, but also how individuals work with the technology. Not all of the video taped sessions were transcribed, however the researcher did review all sessions to ensure that the segments that were transcribed did constitute a representative and unbiased sample.

The video analysis was approached from several standpoints; practical, physical, theoretical, and temporal (Lesh & Lehrer, 2000). From a practical point, the researcher initially transcribed segments of the video taped sessions. Although these sessions are sometimes referred to as lessons, each video did not capture an entire actual lesson. Therefore a lesson is referred to as meaning the actual taped product that was captured during a class period. After these sessions were transcribed, the researcher reviewed all sessions to see if there were any omissions or biased interpretations that may have occurred. The researcher used cross referencing with student work, teacher log, and student interviews to ensure consistency of information. The researcher also looked for generalizations that occurred in collaborative groups using CAS technology over time.

Unfortunately, due to physical and technology restraints there was only one stationary video recording devise per classroom. The researcher looked at the videotapes from a problem solving perspective to look at how students work together on these directed learning activities involving this technology. Although the intent of the video was to look for the social process of the learning environment, upon review of the video taped lessons, there was no longitudinal data available to comment on the social nature of the learning environment. So although there was some data on students working together, there was no information available to look at students' growth in cooperative groups over time.

From a second perspective the researcher looked at a minute to minute account to look at students work on task to see how students attached a problem as well as how they utilized the technology. Initially students were hesitant to use the technology without instruction. However, most students learned to use the technology without prompts, and those who did not would often consult a group member for help or clarification.

In order to address the phenomenographical component of this research and address the "product" of the TI-Nspire CAS, the researcher reviewed the data sources with the lens of a phenomenographer. Questions that helped gain this focus were; "How do my students use the technology? ", "What statements do my students make directly about the technology or the use of the technology?", "Are students able to use the technology to help them answer questions?". These questions helped the researcher to be able to analyze CAS as a phenomenon.

In order to look at and analyze the data through the lens of instrumental genesis, the data was reviewed longitudinally to look at changes over time. In terms of the student-tool relationship, the tool referring to the TI-Nspire CAS, although the activities the students completed in cooperative groups were designed primarily as exploratory activities, the researcher took note of student use of the TI-Nspire CAS as a tool and looked for individual cases of students use extending from merely exploratory to expressive. The term expressive activity is referring to the variety of activities and approaches that students produce when left independently to solve a problem with little or no guidance (Zbiek et al., 2007). Specifically, the data was reviewed looking for specific examples which demonstrated not only a student's ability to use the TI-Nspire CAS independently, but which also showed how the student's understanding had been shaped by the TI-Nspire CAS as a tool. The sequential reviewing of data was important to look for these patterns of development of this ability to learn from and use the technology.

Additional data was gathered from students who participated in the focus group interviews. "Although interviews can be viewed as strategic and politically situated rituals" (Gubrium & Koro-Ljungberg, 2005, p.113){NOTE:p.113} the intention of these small focus group interviews was to allow students to express their perspective on whether the use of CAS had helped them learn mathematics. In order to analyze these audio taped interviews, the

researcher transcribed each of the interviews. The interviews were then be coded to look for

emerging themes. Using a constant comparative method, the researcher cross referenced the data

from the interviews with video taped transcribed sessions, student work, and the teacher log to

support student statements and development of an emerging theory (Charmaz, 2006).

As in a teaching experiment, the researcher started analysis while collecting data. The

goal of doing this was to see trends and create activities that would address a need or issue that

appeared in a previous lesson. The teacher log as well as student work were used to make these

midway revisions. The video tapes were not analyzed until completion of the study period.

Overall, the goal was that the data collect and analyzed would give the researcher some

useful information about student use of this technology and its effects on at least some aspect of

students learning of mathematics. The intent of this pilot study was to gain insight into student

use of this technology. This pilot study gave the researcher ideas about topics which can be

studied using CAS technology as well as methods appropriate for relevant data collection. The

data collected has helped to inform the researcher in designing a larger scale dissertation research

project in which mixed methods research with a cross over design is employed.

Conclusions

With respect to my first research question "what are the effects of the use of the TI-

Nspire CAS on student learning of mathematics. Most students felt that using the TI-Nspire CAS

helped them to better understand the steps in solving an equation. This was evidenced not only

during students during student interviews, but it was also witnessed in video recordings and

teacher logs. When asked how to solve $\frac{6}{5} = \frac{x+3}{8}$ Sally said "multiply by 5 and then multiply by

8. Maggie replied "Don't you just cross multiply?" I had all students type the equation in their calculator and asked them to multiply by 5 then multiplied by 8. I asked Maggie to "cross multiply" I had to remind her to put parentheses on the x+3. Students saw what we had after we multiplied both sides by 5 and 8 and realized it was the same as when we had cross multiplied. They had not previously realized that they were the same thing. Students had been 'taught' to cross multiply, but had not ever realized why it worked and had never linked it with performing equal operations to both sides of the equation. Using the TI-Nspire CAS enabled students to see each step in solving and allowed them to work towards isolating the variable in solving equations. In fact for one student who had always used back solving to try to find a solution, using the TI-Nspire CAS to work backwards through the order of operation was what made the light for him finally go on. "Oh, is that all I have to do!?" he exclaimed as he 'undid' operations to get the variable by itself.

When students worked on problems by themselves, I asked them after they entered the equation into the calculator to double check that it looked like their original equation. For example, when asked to solve $\dfrac{6}{5} = \dfrac{x+3}{8}$, when students typed in the equation they typed

6/5=x+3/8 and when they pressed enter their calculator said $\dfrac{6}{5} = x + \dfrac{3}{8}$. Although some students continued not realizing the equation did not look like their original, most students did notice and were able to go back and figure out they had to go back and put in a parenthesis without prompting. Thus, the way the information was actually displayed helped students to realize that parentheses were needed. However, due to the nature of this study, I was unable to ascertain whether students' understanding of the fact there needed to be parentheses tied into their understanding of the order of operations.

Parenthetical use was also noticed in another activity. When typing $\left(x^3\right)^2$, some students

received the answer of x^9 and others received x^6. Students were confused as they initially

compared what they typed in and saw the two as the same, but upon closer inspection, they

realized that the error was that the second parenthesis was only in the exponent. They had typed

$(x^{3)2}$ and should have typed $\left(x^3\right)^2$. I found that group members remembered to look at the

display closely for this type of error for trouble shooting when group answers were not

consistent.

Another area of solving that has always been problematic for my students is solving an

equation for a given variable. Although my students could tell you that you were trying to isolate

the y for example, they were often unsure what they were supposed to do first in trying to "undo"

operations. For example, when asked to solve $2x + 3y = 8$ for y, many students initially did not

know what to do. I asked them to use the calculator and "try something." Jose' subtracted x and

Received $x + 3y = 8 - x$. He then said "I guess I have to subtract another x." He tried this and was

then was able to complete the problem with the aid of the technology, but with no further

assistance from the teacher. Thus, although initially all students were confused by or refused to

do these types of problems, with the use of the TI-Nspire CAS, most students learned how to

manipulate the symbols to solve for the needed variable.

When using the TI-Nspire to solve equations, several students commented that they liked

how easy it was to go back if an operation they had tried did not appear to make things look

more simplified. Most students said something about being able to "see" things more clearly due

to the screen size and "pretty print." By "seeing" things better students referred to the actual text

size and font as well as the way expressions were displayed on the screen itself. For example it

would retain the "look" of the original problem when you typed it in as in the previous example

$\frac{6}{5} = \frac{x+3}{8}$ so you would know if you had typed something incorrectly. Also, teacher log and

video analysis confirm cases where students arrived at the same answer in different ways and

were okay with their being more than one right way to solve a problem. "You got the same

answer – it just took you more steps," Sally said to a member of her group. Many students no

longer had the "there is only one way to do it" mentality. Although hey realized the solution

itself was either right or wrong, they appeared to accept that there were different ways to arrive

at that solution.

     Also, because it would automatically simplify, it helped students see that multiplying by

½ and dividing by 2 were the same thing. In a problem, students were given the area of a triangle

and asked to find the length of the base of the triangle when the height was 7cm. Students

plugged the value h=7 into the formula $A = \frac{1}{2}bh$. However, doing this by hand students got

$28 = \frac{1}{2}\Box b\Box 7$ and did not know what to do. Those students that chose to use the technology typed

the same equation into the TI-Nspire CAS and the display read $28 = \frac{7\Box b}{2}$. These students had no

difficulty completing the problem. Initially there was some confusion as Maggie asked, "where

did the ½ go?" However, as soon as she finished the sentence she realized, "oh, it is there,

divided by two is the same thing."

     This same idea of the way problems are displayed also came up in an activity involving

trapezoids. On an activity where I had students find the area of a trapezoid with height h and

base lengths b and c by breaking it up into two triangles, students were able to express the areas

of each of the triangles, but were not sure what to do as they did not understand what it meant to

add $\frac{1}{2}bh + \frac{1}{2}ch$. "How can we add them, we don't have numbers?, " Jennifer asked. I

encouraged them to use the TI-Nspire CAS to try to simplify the expression. Typing this in gave

them $\frac{bh}{2} + \frac{ch}{2}$. I then asked them to factor out any common terms and explain what their result

meant in "real" terms. $Area = \frac{h}{2}(b+c)$ is what students arrived at, and most all were able to

write out what it meant in words; "the area of a trapezoid is equal to one half the height times the

sum of the bases" or something to that effect. Sheila even exclaimed: "Oh my god, it's a

formula!"

Although some students resisted using the TI-Nspire CAS. (" Why do we need to learn

how to do it on this if we can do it by hand.") Most students came to feel comfortable using the

CAS technology. Not all students signed out a TI-Nspire CAS calculator for the year, which I

feel made a difference in the mastery level obtained by the students. All students who signed out

this calculator for the year said that they felt comfortable using it. In fact a few students said that

it became their "calculator of choice."

Unfortunately, due to the limited audio range of the single camera and microphone, I was

not able to find data which could allow me to comment on the impact of the design of the

learning environment with respect to student learning. Student work was always completed in

cooperative group settings, and students demonstrated continual ability to help each other trouble

shoot and explain their answers, though not always using the most accurate use of the

terminology in a formal sentence structure. Therefore, the observations by the teacher as well as

student work and video tape suggest that working in cooperative groups was beneficial to student understanding of concepts.

Overall, most students were able to use the technology to complete guided learning activities in their groups. Although some students were hesitant about using the technology, when they would get stuck using by hand methods, they would ask group members who were more comfortable with the TI-Nspire CAS for help. Most students did like the way it made things look. In fact during one group interview students commented on how they felt that using this calculator to solve geometry problems involving surface area and volume, actually helped them to memorize the formulas. When asked how, Jose' replied, "when you type it in again and again, it keeps it the way you plugged it in, so you can see it that way and remember it."

From the data I have reviewed I have seen supporting evidence that using the TI-Nspire CAS has had a positive effect on my students understanding of solving equations, using parentheses, and understanding equivalent operations (i.e. dividing by 2 and multiplying by ½). Therefore, it does have a positive effect on student learning of algebraic rules and symbolic manipulation for at least some topics. Several students did comment on how using the TI-Nspire CAS helped them to understand the procedures used in solving an equation, or solving for a specific variable in an equation. However, I never did test retention of these skills and procedures without a calculator. To what extent it has helped and whether their knowledge has reached that of a conceptual level has not been clearly demonstrated through the data I have collected.

Further research

This pilot study did offer some possible areas to further research. For example, students appeared to notice the importance of parentheses. However, although there was some evidence to support this, this qualitative research study has not adequately tested the students' knowledge or understanding of the order of operations. There was no pretest given in order to compare post test grades. Another concept that was supported by student work, teacher log, video taped lessons, and student interviews was the ability for students to understand how to solve an equation. One way to test to see if students can solve equations for a given variable and determine if they understand the order of operations might be to give a calculator free pre and post test using nonsense equations and ask students to indicate each step they used to solve for the given variable. This skill could be practiced as well by backwards design. Students could create their own formula by starting off with one variable and one step at a time perform some operation to it to get a formula writing down each step. The formula could have at least 3 variables. Students would then have to explain how they would step by step go backwards to get their formula solved for the variable they started with.

Much of the fear and hesitation of CAS technology is due to the way it is perceived as a black box- a device that spits out answers without understanding (Drijvers, 2000). When I say that students used the CAS as a symbolic manipulator, one may see this as a strictly procedural tool. However, I would argue that using a symbolic manipulator as a tool to carry out symbolic manipulative procedures does not only help students to become user-agents, hence learn the procedural knowledge necessary to complete by hand calculations, but that by using a dynamic tool to mentally and physically see and understand manipulations it has the potential to further a students conceptual knowledge of mathematical concepts (Baroody, Feil, & Johnson, 2007; Star, 2005, 2007). I see this as a great opportunity to explore procedural knowledge and what the

effects of gaining procedural fluency through the use of CAS are with respect to student learning of mathematics as well as whether there is any effect on student understanding of mathematical concepts and their overall conceptual knowledge. From conducting this pilot study I realize that in order to look at the acquisition and retention of procedural knowledge through the use of the TI-Nspire CAS as well as the conversion of this knowledge to conceptual knowledge that I will need to give pre and post tests with carefully written problems that will address these areas. I then will have data that will be quantifiable and a study that could be replicated.

Also, as a lot of the research on the use of CAS in algebra is on College Algebra, I would be interested in looking at using CAS with students as they are first introduce symbolically to algebra (Pre-Alebra). As I do believe that the use of CAS does have the potential to not only help understanding and retention of procedural knowledge, but also conceptual understanding as well. Again a pre and post test designed experiment could help to see and quantify finding that would support this theory.

In this pilot study I hoped to look at the design of the class and how that affected student understanding, but I did not have the controls I needed for comparison with the data I collected. Therefore a study which included a cross over design would control for students using the technology versus not using the technology and hopefully allow me to look at the CAS effect on student work in a group setting.

Another issue I have thought about pertains to the population of students. I did not specifically separate out differences based on ethnicity or sex, but in a larger scale study, I do think it would be interesting to look at any issues emerging from any subset of the data due to: gender, ethnicity, English language proficiency, or access to school special education services.

Using stratified purposeful random sampling would allow the researcher to narrow the focus of

the research specific to any of these special interest groups and possibly get a more specific

understanding of the population represented (LeCompte, Preissle, & Tesch, 1993; Patton, 2002)

and how the use of the TI-Nspire CAS could effect these subgroups.

References

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternate reconceptualization of procedural and conceptual knowledge.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis.* Newbury Park: Sage.

Denzin, N. K., & Lincoln, Y. S. (Eds.). (2002). *The handbook of qualitative research (3rd ed.).* Newbury Park: Sage.

Drijvers, P. (2000). Students encountering obstacles using a CAS. *International Journal of Computers for Mathematical Learning, 5*(3), 189-209.

Gubrium, E., & Koro-Ljungberg, M. (2005). Contending with border making in the social constructivist interview. *Qualitative Inquiry, 11*(5), 695.

Heid, M. K. (2002). Computer algebra systems in secondary mathematics classes: The time to act is now! *Mathematics Teacher, 95*(8), 662-667.

Howe, K. R. (1998). The interpretive turn and the new debate in education. *Educational Researcher, 27*(8), 13-20.

LeCompte, M., Preissle, J., & Tesch, R. (1993). *Ethnography and qualitative design in educational research*. San Diego: Academic Press.

Leinbach, C. (2001). Using a CAS in the Mathematics Classroom: Two Examples for Exploring Cubic Curves. *The International Journal of Computer Algebra in Mathematics Education., 8*(2), 131.

Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analyses of conceptual change. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp. 665-708). Mahwah, NJ: Lawrence Erlbaum Associates.

Noguera, N. (2001). A Description of Tenth Grade Algebra Students' Attitudes and Cognitive Development When Learning Algebra Using CAS. *The International Journal of Computer Algebra in Mathematics Education, 8*(4), 257-270.

Patton, M. Q. (2002). *Qualitative research and evaluation methods (3rd ed.)*. Newbury Park: Sage.

Roschelle, J., & Jackiw, N. (2000). Technology design as educational research: Interweaving imagination, inquiry, and impact. In A. E. Kelly & R. A. Lesh (Eds.), *Hanbook of research design in mathematics and science education.* (pp. 777-798). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Star, J. R. (2005). Research commentary: Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Star, J. R. (2007). Foregrounding procedural knowledge. *Journal for Research in Mathematics Education, 38*(2), 132-135.

Vygotsky, L. S. (1986). *Thought and language*. Cambridge, MA: MIT Press.

Zbiek, R. M., Heid, K. M., Blume, G. M., & Dick, T. P. (2007). Research on technology in

mathematics education: A perspective of constructs. In F. Lester (Ed.), *Second handbook

of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC:

Information Age.

Appendix A

The following is a list of interview questions for group interviews:

"This interview is to get feedback from you as to how using the TI-Nspire CAS helped or did not help you understand mathematics."

1. What do you think of the TI-Nspire CAS?

2. Was the TI-Nspire CAS easy to learn how to use?

3. How comfortable do you feel using the TI-Nspire CAS?

4. Can you describe some of the activities you have used the TI-Nspire CAS for?

5. How useful do you feel the TI-Nspire CAS was in helping you understand the material you were studying?

6. What are some of the things you were able to do using the TI-Nspire CAS that helped you to remember a particular concept.

7. Overall, would you recommend using the TI-Nspire CAS with other classes?

8. What suggestions would you give to students learning to use this technology?

A p p e n d i x   B

G E O R G E   M A S O N   H I G H   S C H O O L
7 1 2 4   L E E S B U R G   P I K E
F A L L S   C H U R C H ,   V A   2 0 0 4 3
P H O N E :   (7 0 3)   2 4 8   –   5 5 0 0   E X T   3 0 5 8
F A X :   (7 0 3)   2 4 8   –   5 5 3 3
E - M A I L : G A N T Z L @ F C C P S . K 1 2 . V A . U S

April 15, 2008

Dear Parents of Algebra/Geometry III Students,

As you know we have had the opportunity to work with a new technology this year, the TI-Nspire CAS. It is clear that some students liked working with this new technology and others did not. Last year when I was working on National Board Certification I realized how much I learned about my own teaching as well as student learning through video taping lessons. Therefore I decided to videotape a few of our class sessions this quarter. Another aspect to our classroom this quarter has been the use of journals to answer questions on activities as well as to discuss understanding of concepts and comfort levels with the technology.

In considering what impact this technology has or has not had on student understanding of mathematics I am planning to analyze this data (videotapes, journals, and student assessments) that has already been collected and see whether my students' understanding of mathematical concepts was affected by the use of CAS. In order to do this I plan to look back at the activities we completed in class and compare my observations from the video tapes with the journal entries and students' scores from the assessments that were given in my class.

I am currently a PhD student at George Mason University. I hope that this analysis will provide guidance for my dissertation. I hope that you and your child will allow me to use the data that I have already collected during my classes for my analysis. Your child's name will not be used in the analysis. Please feel free to contact me if you have any questions at (703) 248-5500 Ext. 3058 or e-mail me at < gantzl@fccps.org >.

I plan to interview a few students to get their impressions of this technology. Therefore, I will be conducting group interviews where students will have the opportunity to tell me what they thought about using this technology. Students who would like to participate in a group interview can check the appropriate box on the permission form attached.

Thank you for your support.

Linda A.G. Gantz
Mathematics Department Leader

Appendix C

The Effects of the TI-Nspire CAS on Student Learning
PARENTAL INFORMED CONSENT FORM

RESEARCH PROCEDURES
This research project is being conducted to determine the effects of the TI-Nspire Computer Algebra System (CAS) on student learning. If you agree to allow your child to participate, you will be giving permission for Mrs. Gantz to use data that was collected during class lessons. This data includes video taped lessons, journal entries, and performance on class assessments. The purpose of using this data is to analyze the effects of this technology on student learning. Your child may also agree to participate in a group interview by checking the appropriate box on his/her permission form.

RISKS
There are no foreseeable risks for participating in this research.

BENEFITS
The benefits to your child include learning how to use the new TI-Nspire CAS technology. Other benefits include assisting in adding to the research base on this new technology.

CONFIDENTIALITY
The data in this study will be confidential. The student journal entries, as well as the video taped lessons, assessment scores, and the audio taped group interviews will be coded so that names will not be made public. Only the researcher will have access to the identification key.

PARTICIPATION
Your child's participation is voluntary, and your child may withdraw from the study at any time and for any reason. If you or your child decide not to participate or if your child withdraws from the study, there is no penalty or loss of benefits to which you or your child are otherwise entitled and your child's grade will not be affected.

CONTACT
This research is being conducted by Linda A. G. Gantz of the Mathematics Department at George Mason High School and a student at George Mason University. She may be reached at (703) 248-5500 ext. 3058 for questions or to report a research-related problem. Her faculty advisor is Dr. Patricia Moyer-Packenham who can be reached at (703) 993-3926. You may contact the George Mason University Office of Research Subject Protections at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT
☐ I have read this form and **agree** to allow my child to participate in this study.
  ☐ Yes, my child WILL participate in a group interview.

  ☐ No, my child will not participate in a group interview.

☐ I have read this form and **do not agree** to allow my child to participate in this study.

_____ **of** _____
Name (Parent/Guardian)   Name of Child

_____
Signature (Parent/Guardian)

_____
Date of Signature   .

Appendix D

The Effects of the TI-Nspire CAS on Student Learning
STUDENT INFORMED ASSENT FORM

RESEARCH PROCEDURES
This research project is being conducted to determine the effects of the TI-Nspire Computer Algebra System (CAS) on student learning. If you agree to participate, you will be giving permission for Mrs. Gantz to use data that was collected during class lessons. This data includes video taped lessons, journal entries, and performance on class assessments. The purpose of using this data is to analyze the effects of this technology on student learning. You may also agree to participate in a group interview by checking the appropriate box below.

RISKS
There are no foreseeable risks for participating in this research.

BENEFITS
The benefits to you include learning how to use the new TI-Nspire CAS technology. Other benefits include assisting in adding to the research base on this new technology.

CONFIDENTIALITY
The data in this study will be confidential. The student journal entries, as well as the video taped lessons, assessment scores, and the audio taped group interviews will be coded so that names will not be made public. Only the researcher will have access to the identification key.

PARTICIPATION
Your participation is voluntary, and you may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled and your grade will not be affected. There are no costs to you or any other party.

CONTACT
This research is being conducted by Linda A. G. Gantz of the Mathematics Department at George Mason High School and a student at George Mason University. She may be reached at (703) 248-5500 ext. 3058 for questions or to report a research-related problem. Her faculty advisor is Dr. Patricia Moyer-Packenham who can be reached at (703) 993-3926. You may contact the George Mason University Office of Research Subject Protections at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.
This research has been reviewed according to George Mason University procedures governing your participation in this research.

ASSENT
☐ I have read this form and **agree** to participate in this study.
　　☐ Yes, I WILL participate in a group interview.

　　☐ No, I will not participate in a group interview.

☐ I have read this form and **do not agree** to participate in this study.

_____

Name (Student)

_____

Signature (Student)

_____

Date of Signature   Version

Appendix E

Algebra   Name:_____

Exponents   Date:_____

I. Zero Exponent Property & Negative Exponent Property

Use the TI-Nspire CAS to fill in the chart below. Leave your final answers as fractions.

2^{-3} = 3^{-3} =

2^{-2} = 3^{-2} =

2^{-1} = 3^{-1} =

2^{0} = 3^{0} =

Complete the following WITHOUT using the calculator.

1. a) Based on the values above, what can you say about the value of 4^{0} ?

   b) What is the value of x^{0}? Is there any exception as to what x can be?

2. a) Based on the values above, what is the value of 4^{-2}?

   b) In general explain what happens when you put a number to a negative

   power? *(You may use an example to help your explanation)*

II. Product of Powers Property

A. Use the TI-Nspire CAS to simplify the following expressions:

Make sure to use a multiplication sign between the terms.

For example #1 you would type: $x \wedge 3$ x $x \wedge 4$

1. $x^3 \cdot x^4$ _____	4. $y^{-3} \cdot y^{-4}$ _____
2. $x \cdot x^7$ _____	5. $x^5 \square x^5$ _____
3. $y^{-4} \cdot y^{21}$ _____	6. $x^2 \cdot x^3$ _____

Using your answers above, find a rule for simplifying exponential expressions.

_____

_____

_____

B. NO CALCULATOR! Test your conjecture by simplifying the following expressions **without** using the calculator.

7. $y^{10} \cdot y^3$ _____	9. $y^2 \cdot y^{-73}$ _____
8. $y^{65} \cdot y^{-13}$ _____	10. $x^{45} \cdot x^{-45}$ _____

III. Power of a Power Property

A. Use the TI-Nspire CAS to simplify the following expressions. Make sure to include the parentheses.

1. $(x^3)^2$ _____	4. $\left(x^{-2}\right)^{-5}$ _____
2. $\left(x^7\right)^7$ _____	5. $\left(x^{-5}\right)^5$ _____
3. $\left(y^{11}\right)^3$ _____	6. $\left(x^{-5}\right)^{-3}$ _____

Using your answers above, find a rule for simplifying exponential expressions.

_____

_____

_____

B. NO CALCULATOR. Test your conjecture by simplifying the following expressions **without** using the calculator.

7. $\left(x^2\right)^3$ _____	9. $\left(x^{-5}\right)^{-2}$ _____
8. $\left(y^{-13}\right)^{-3}$ _____	10. $\left(y^{12}\right)^{-12}$ _____

IV. Power of a Product Property

A. Use the TI-Nspire CAS to simplify the following expressions:

1. $\left(3xy^{-1}\right)^3$ _____	4. $\left(3xy^2z^{-5}\right)^{-2}$ _____
2. $\left(2y^{-13}\right)^{-3}$ _____	5. $\left(3xy^2z^{-5}\right)^2$ _____
3. $\left(xy^{42}\right)^5$ _____	6. $(ax^by^c)^d$ _____

Using your answers above, find a rule for simplifying exponential expressions.

_____

_____

_____

B. NO CALCULATOR. Test your conjecture by simplifying the following expressions **without** using the calculator.

7. $\left(xy^{-3}\right)^{-7}$ _____	9. $(x^2y^3z^{-2})^5$ _____
8. $\left(2x^5yz^5\right)^3$ _____	10. $(4x^3yz)^2$ _____

V. Quotient of Powers Property

A. Use the TI-Nspire CAS to simplify the following expressions:

1. $\dfrac{2x^2}{x^3}$ _____	4. $\dfrac{9x^{-3}}{x^5}$ _____
2. $\dfrac{3x^8}{y^8}$ _____	5. $\dfrac{y^5}{2y^{-7}}$ _____
3. $\dfrac{5y^{11}}{y^2}$ _____	6. $\dfrac{3x^3}{4x^5}$ _____

Using your answers above, find a rule for simplifying exponential expressions.

_____

_____

_____

B. NO CALCULATOR. Test your conjecture by simplifying the following expressions **without** using the calculator.

7. $\dfrac{2x^3}{x^{-12}}$ _____	9. $\dfrac{x^{25}}{x^{25}}$ _____
`8. $\dfrac{3y^5}{y}$ _____	10. $\dfrac{y^{-7}}{y^7}$ _____

VI. Power of a Quotient Property

A. Use the TI-Nspire CAS to simplify the following expressions:

1. $\left(\dfrac{x}{y}\right)^5$ _____	4. $\left(\dfrac{x^4}{y^4}\right)^{-7}$ _____
2. $\left(\dfrac{x}{y^2}\right)^2$ _____	5. $\left(\dfrac{y^{-5}}{x^{-7}}\right)^5$ _____
3. $\left(\dfrac{2x^{-2}}{y^4}\right)^{-3}$ _____	6. $\left(\dfrac{2x^3}{3y}\right)^2$ _____

Using your answers above, find a rule for simplifying exponential expressions.

_____

_____

_____

B. NO CALCULATOR. Test your conjecture by simplifying the following expressions **without** using the calculator.

7. $\left(\dfrac{x}{y^7}\right)^8$ _____	9. $\left(\dfrac{x^{-3}}{y^{17}}\right)^{-2}$ _____
`8. $\left(\dfrac{y^3}{x^3}\right)^4$ _____	10. $\left(\dfrac{3x^{-7}}{y^2}\right)^{-2}$ _____

Appendix F

Algebra   Name:_____

Solving for a given variable   Date:_____

Use the TI-Nspire CAS to solve for the variable indicated. Make sure that when you type in the original equation you use a multiplication sign between each variable. Example: for A = bh you would type A = b x h

1. s = 180 (n – 2)	
Solve for n	
Explain each step	
s = 180 (n – 2)	
n =	
2. $e = \dfrac{360}{n}$	

Solve for n

Explain step by step

$e = \dfrac{360}{n}$	

n =

3. $A = \dfrac{1}{2} \, b \cdot h$

Solve for h

Explain each step

$a = \dfrac{1}{2} \, b \cdot h$	

h =

4. $A = \frac{1}{2} b \cdot h$

Solve for b

Explain each step

$a = \frac{1}{2} b \cdot h$	

b =

5. $A = \frac{1}{2} (b + c) \cdot h$

Solve for h

Explain each step

$a = \frac{1}{2} (b + c) \cdot h$	

h=

6. $A = \frac{1}{2}$ (b + c) •h

Solve for b

Explain each step

$a = \frac{1}{2}$ (b + c) •h	

b =

Appendix G

Alg/Geom III   Name:_____

                                Date:_____

Equivalent Expressions

You may use the TI-Nspire CAS to help you to write two equivalent expressions for the expression written below.

Definition: **Equivalent fractions** are _____

_____

Original Expression	Equivalent Expression #1	Equivalent Expression #2
$\frac{1}{2}(a+b)$		
$\frac{1}{3}(3c+d)$		
$b\square\frac{1}{2}\square h$		

$(x)(y)\left(\dfrac{1}{2}\right)$		

Why is it important to be able to write and recognize equivalent expressions?

_____
_____
_____